THE DELTA-SIGMA TOOLBOX Version 7.1

Getting Started

The Delta-Sigma toolbox requires the `Signal Processing Toolbox` and the `Control Systems Toolbox`. Certain functions (`clans` and `designLCBP`) also require the `Optimization Toolbox`.

To obtain a copy of the Delta-Sigma toolbox, go to the MathWorks web site (`http://www.mathworks.com/matlabcentral/fileexchange`), find the `Controls` category and select `delsig`. To improve simulation speed, compile the `simulateDSM.c` file by typing `mex simulateDSM.c` at the Matlab prompt. Do the same for `simulateESL.c` and `ai2mif.c`.

For more detailed descriptions of selected functions and more examples, see Chapters 8 and 9 of

[1] R. Schreier and G. C. Temes, *Understanding Delta-Sigma Data Converters*, John Wiley & Sons, New York, 2004.

Toolbox Conventions

Frequencies are normalized; $f = 1$ corresponds to f_s.

Default values for function arguments are shown following an equals sign (=) in the parameter list. To use the default value for an argument, omit the argument if it is at the end of the list, otherwise use `NaN` (not-a-number) or `[]` (the empty matrix) as a place-holder.

A matrix is used to describe the loop filter of a general single-quantizer delta-sigma modulator. See "MODULATOR MODEL DETAILS" on page 20 for a description of this "ABCD" matrix.

Demonstrations and Examples

`dsdemo1`	Demonstration of the `synthesizeNTF` function. Noise transfer function synthesis for a 5th-order lowpass modulator, both with and without optimized zeros, plus an 8th-order bandpass modulator with optimized zeros.
`dsdemo2`	Demonstration of the `simulateDSM`, `predictSNR` and `simulateSNR` functions: time-domain simulation, SNR prediction using the describing function method of Ardalan and Paulos, spectral analysis and signal-to-noise ratio. Lowpass, bandpass, multi-bit lowpass examples are given.
`dsdemo3`	Demonstration of the `realizeNTF`, `stuffABCD`, `scaleABCD` and `mapABCD` functions: coefficient calculation and dynamic range scaling.
`dsdemo4`	Audio demonstration of MOD1 and MOD2 with sinc^n decimation.
`dsdemo5`	Demonstration of the `simulateESL` function: simulation of the element selection logic of a mismatch-shaping DAC.
`dsdemo6`	Demonstration of the `designHBF` function. Hardware-efficient halfband filter design and simulation.
`dsdemo7`	Demonstration of the `findPIS` function: positively-invariant set computation.
`dsdemo8`	Demonstration of the `designLCBP` function: continuous-time bandpass modulator design. (This function requires the `Optimization Toolbox`.)
`dsexample1`	Discrete-time lowpass modulator design.
`dsexample2`	Discrete-time bandpass modulator design.

KEY FUNCTIONS

`ntf = synthesizeNTF(order=3,R=64,opt=0,H_inf=1.5,f0=0)` page 4
`ntf = clans(order=4,R=64,Q=5,rmax=0.95,opt=0)` page 5

Synthesize a noise transfer function.

`[snr,amp,k0,k1,sigma_e2] = predictSNR(ntf,R=64,amp=...,f0=0)` page 6

Predict the SNR vs. input power curve using the describing function method of Ardalan and Paulos.

`[v,xn,xmax,y] = simulateDSM(u,ABCD,nlev=2,x0=0)` page 7
`[v,xn,xmax,y] = simulateDSM(u,ntf,nlev=2,x0=0)`

Simulate a delta-sigma modulator with a given input.

`[snr,amp] = simulateSNR(ntf,R,amp=...,f0=0,nlev=2,f=1/(4*R),k=13)` page 8

Determine the SNR vs. input power curve by simulation.

`[a,g,b,c] = realizeNTF(ntf,form='CRFB',stf=1)` page 9

Convert a noise transfer function into coefficients for a specific structure.

`ABCD = stuffABCD(a,g,b,c,form='CRFB')` page 10

Calculate the ABCD matrix given the parameters of a specified modulator topology.

`[a,g,b,c] = mapABCD(ABCD,form='CRFB')` page 10

Calculate the parameters of a specified modulator topology given the ABCD matrix.

`[ABCDs, umax] = scaleABCD(ABCD,nlev=2,f=0,xlim=1,ymax=nlev+2))` page 11

Perform dynamic range scaling on a delta-sigma modulator described by ABCD.

`[ntf,stf] = calculateTF(ABCD,k=1)` page 12

Calculate the NTF and STF of a delta-sigma modulator described by the ABCD matrix, assuming a quantizer gain of k.

`[sv,sx,sigma_se,max_sx,max_sy]=simulateESL(v,mtf,M=16,dw=[1…],sx0=[0…])` page 13

Simulate the element-selection logic in a mismatch-shaping DAC.

`[f1,f2,info] = designHBF(fp=0.2,delta=1e-5,debug=0)` page 14

Design a hardware-efficient half-band filter for use in a decimation or interpolation filter.

`[param,H,L0,ABCD,x] = designLCBP(n=3,OSR=64,opt=2,Hinf=1.6,`
`f0=1/4,t=[0 1],form='FB',x0,dbg)` page 17

Design a continuous-time LC-bandpass modulator.

`[s,e,n,o,Sc] = findPIS(u,ABCD,nlev=2,options)` page 19

Find a convex positively-invariant set for a delta-sigma modulator.

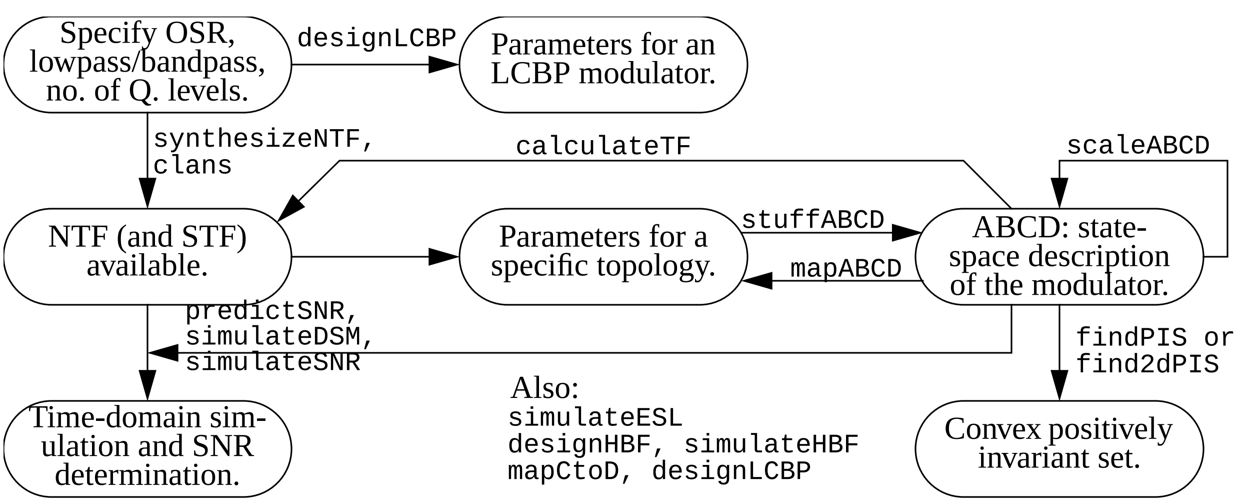


Figure 1: Operations performed by the basic commands.

OTHER SELECTED FUNCTIONS

Delta-Sigma Utility

`mod1, mod2`

Scripts for setting up the ABCD matrix, NTF and STF of the 1st/2nd-order modulator.

`snr = calculateSNR(hwfft,f)`

Estimate the SNR given the in-band bins of a Hann-windowed FFT and the location of the input signal.

`[sys, Gp] = mapCtoD(sys_c,t=[0 1],f0=0)`

Map a continuous-time system to a discrete-time system whose impulse response matches the sampled pulse response of the original continuous-time system.

`[A B C D] = partitionABCD(ABCD, m)`

Partition ABCD into A, B, C, D for an *m*-input state-space system.

`H_inf = infnorm(H)`

Compute the infinity norm (maximum absolute value) of a z-domain transfer function. See `evalTF`.

`sigma_H = rmsGain(H,f1,f2)`

Compute the root mean-square gain of the discrete-time transfer function H in the frequency band (`f1`,`f2`).

General Utility

`dbv(), dbp(), undbv(), undbp(), dbm()`

The dB equivalent of voltage/power quantities, and their inverse functions.

`window = hann(N)`

A Hann window of length `N`. Unlike MATLAB's original `hanning` function, `hann` does not smear tones which are located exactly in an FFT bin (i.e. tones having an integral number of cycles in the given block of data). MATLAB 6's `hanning(N,'periodic')` function is the same as `hann(N)`.

Graphing

`plotPZ(H,color='b',markersize=5,list=0)`

Plot the poles and zeros of a transfer function.

`figureMagic(xRange,dx,xLab, yRange,dy,yLab, size)`

Performs a number of formatting operations for the current figure, including axis limits, ticks and labelling.

`printmif(file,size,font,fig)`

Print graph to an Adobe Illustrator file and then use `ai2mif` to convert it to FrameMaker MIF format. `ai2mif` is an improved version of the function of the same name originally written by Deron Jackson <djackson@mit.edu>.

`[f,p] = logsmooth(X,inBin,nbin)`

Smooth the fft, X, and convert it to dB. See also `bplogsmooth` and `bilogplot`.

synthesizeNTF

Synopsis: `ntf = synthesizeNTF(order=3,OSR=64,opt=0,H_inf=1.5,f0=0)`
Synthesize a noise transfer function (NTF) for a delta-sigma modulator.

Arguments

order	The order of the NTF. *order* must be even for bandpass modulators.
OSR	The oversampling ratio. *OSR* is only needed when optimized NTF zeros are requested.
opt	A flag used to request optimized NTF zeros. *opt*=0 puts all NTF zeros at band-center (DC for lowpass modulators). *opt*=1 optimizes the NTF zeros. For even-order modulators, *opt*=2 puts two zeros at band-center, but optimizes the rest.
H_inf	The maximum out-of-band gain of the NTF. Lee's rule states that *H_inf*<2 should yield a stable modulator with a binary quantizer. Reducing *H_inf* increases the likelihood of success, but reduces the magnitude of the attenuation provided by the NTF and thus the theoretical resolution of the modulator.
f0	The center frequency of the modulator. *f0*≠0 yields a bandpass modulator; *f0*=0.25 puts the center frequency at $f_s/4$.

Output

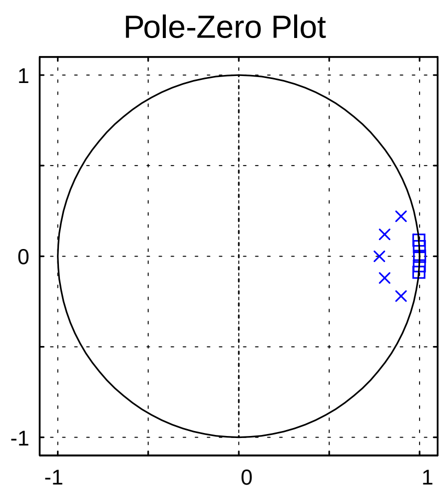
ntf	The modulator NTF, given as an LTI object in zero-pole form.

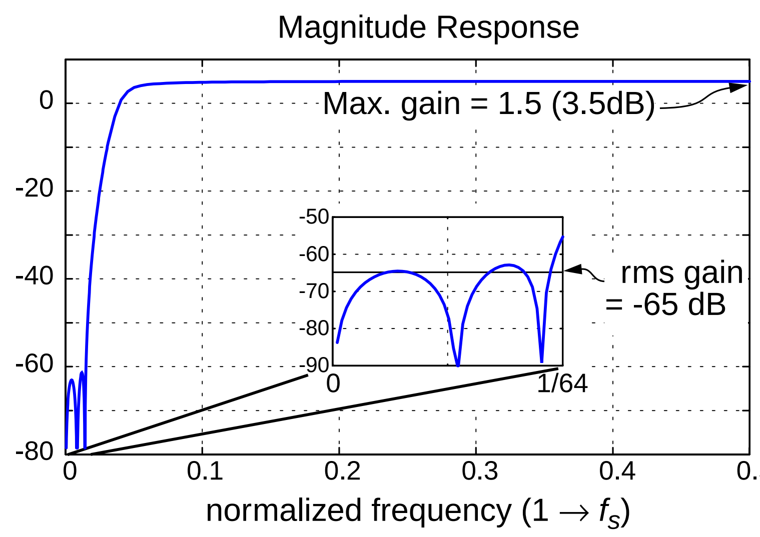
Example

Fifth-order lowpass modulator; zeros optimized for an oversampling ratio of 32.

```
>> H = synthesizeNTF(5,32,1)
Zero/pole/gain:
       (z-1) (z^2 - 1.997z + 1) (z^2 - 1.992z + 1)
----------------------------------------------------------
(z-0.7778) (z^2 - 1.613z + 0.6649) (z^2 - 1.796z + 0.8549)

Sampling time: 1
```





clans

Synopsis: `ntf = clans(order=4,OSR=64,Q=5,rmax=0.95,opt=0)`
Synthesize a noise transfer function (NTF) for a lowpass delta-sigma modulator using the CLANS (Closed-loop analysis of noise-shaper) methodology [1]. This function requires the optimization toolbox.

[1] J. G. Kenney and L. R. Carley, "Design of multibit noise-shaping data converters," *Analog Integrated Circuits Signal Processing Journal*, vol. 3, pp. 259-272, 1993.

Arguments

order	The order of the NTF.
OSR	The oversampling ratio.
Q	The maximum number of quantization levels used by the fed-back quantization noise. (Mathematically, $Q = \|h\|_1 - 1$, i.e. the sum of the absolute values of the impulse response samples minus 1, is the maximum instantaneous noise gain.)
rmax	The maximum radius for the NTF poles.
opt	A flag used to request optimized NTF zeros. *opt*=0 puts all NTF zeros at band-center (DC for lowpass modulators). *opt*=1 optimizes the NTF zeros. For even-order modulators, *opt*=2 puts two zeros at band-center, but optimizes the rest.

Output

ntf	The modulator NTF, given as an LTI object in zero-pole form.

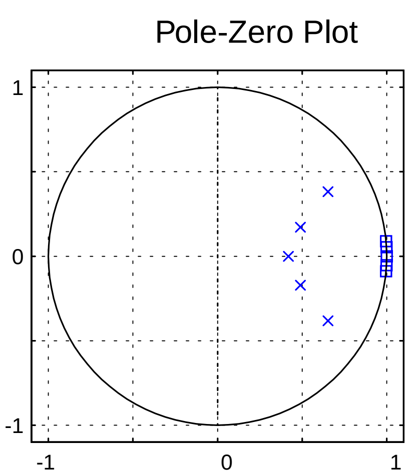
Example

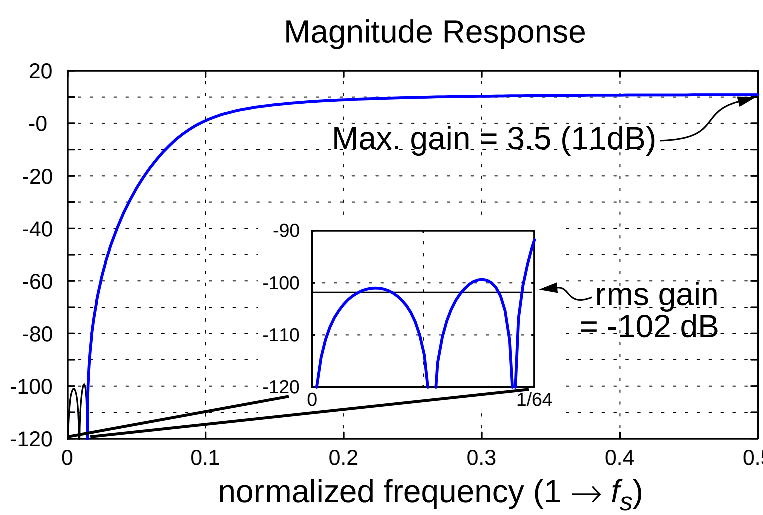
Fifth-order lowpass modulator; (time-domain) noise gain of 5, zeros optimized for *OSR* = 32.

```
>> H= clans(5,32,5,.95,1)

Zero/pole/gain:
       (z-1) (z^2 - 1.997z + 1) (z^2 - 1.992z + 1)
----------------------------------------------------------
(z-0.4183) (z^2 - 0.9784z + 0.2685) (z^2 - 1.305z + 0.5714)

Sampling time: 1
```





predictSNR

Synopsis: `[snr,amp,k0,k1,sigma_e2] = predictSNR(ntf,OSR=64,amp=...,f0=0)`
Use the describing function method of Ardalan and Paulos [1] to predict the signal-to-noise ratio (SNR) in dB for various input amplitudes. This method is only applicable to binary modulators.

[1] S. H. Ardalan and J. J. Paulos, “Analysis of nonlinear behavior in delta-sigma modulators,” *IEEE Transactions on Circuits and Systems*, vol. 34, pp. 593-603, June 1987.

Arguments

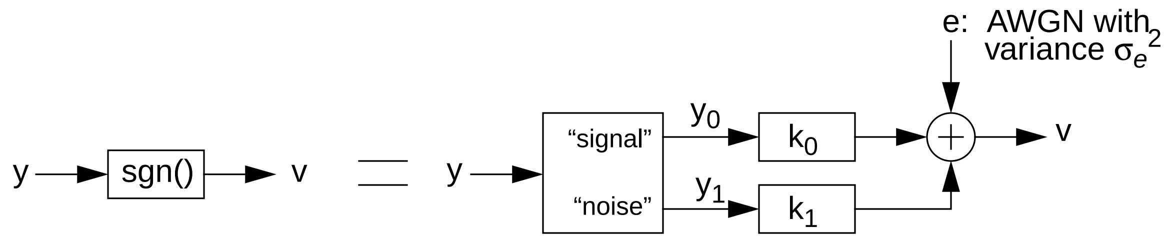
ntf	The modulator NTF, given in zero-pole form.
OSR	The oversampling ratio. *OSR* is used to define the “band of interest.”
amp	A row vector listing the amplitudes to use. Defaults to [-120 -110...-20 -15 -10 -9 -8 … 0] dB, where 0 dB means a full-scale (peak value = 1) sine wave.
f0	The center frequency of the modulator. $f0 \neq 0$ corresponds to a bandpass modulator.

Output

snr	A row vector containing the predicted SNRs.
amp	A row vector listing the amplitudes used.
k0	The signal gain of the quantizer model; one value per input level.
k1	The noise gain of the quantizer model; one value per input level.
sigma_e2	The mean square value of the noise in the model of the quantizer.

Example
See the example on page 8.

The Quantizer Model:
The binary quantizer is modeled as a pair of linear gains and a noise source, as shown in the figure below. The input to the quantizer is divided into signal and noise components which are processed by signal-dependent gains k_0 and k_1. These signals are added to a noise source, which is assumed to be white and to have a Gaussian distribution (the variance σ_e^2 is also signal-dependent), to produce the quantizer output.



simulateDSM

Synopsis: `[v,xn,xmax,y] = simulateDSM(u,ABCD,nlev=2,x0=0)` or
`[v,xn,xmax,y] = simulateDSM(u,ntf,nlev=2,x0=0)`

Simulate a delta-sigma modulator with a given input. For maximum speed, make sure that the `mex` file is on your search path (At the MATLAB prompt, type `which simulateDSM`).

Arguments

u	The input sequence to the modulator, given as a $m \times N$ row vector. m is the number of inputs (usually 1). Full-scale corresponds to an input of magnitude *nlev*–1.
ABCD	A state-space description of the modulator loop filter.
ntf	The modulator NTF, given in zero-pole form. The modulator STF is assumed to be unity.
nlev	The number of levels in the quantizer. Multiple quantizers are indicated by making *nlev* an array.
x0	The initial state of the modulator.

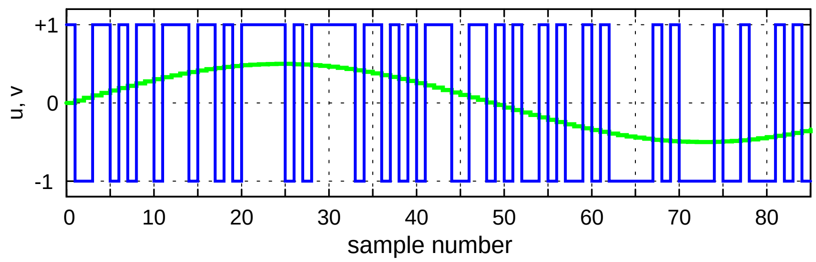
Output

v	The samples of the output of the modulator, one for each input sample.
xn	The internal states of the modulator, one for each input sample, given as an $n \times N$ matrix.
xmax	The maximum absolute values of each state variable.
y	The samples of the quantizer input, one per input sample.

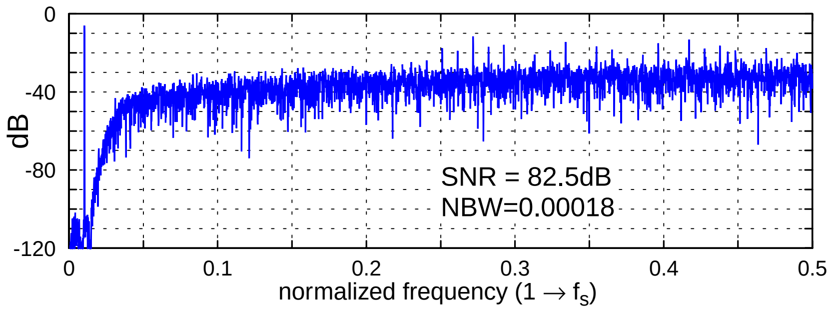
Example

Simulate a 5th-order binary modulator with a half-scale sine-wave input and plot its output in the time and frequency domains.

```
>> OSR = 32; H = synthesizeNTF(5,OSR,1);
>> N = 8192; fB = ceil(N/(2*OSR)); f=85; u = 0.5*sin(2*pi*f/N*[0:N-1]);
>> v = simulateDSM(u,H);
```




```
t = 0:85;
stairs(t, u(t+1));
hold on;
stairs(t,v(t+1));
axis([0 85 -1.2 1.2]);
ylabel('u, v');
```




```
spec=fft(v.*hann(N))/(N/4);
plot(linspace(0,1,N/2),
dbv(spec(1:N/2)))
axis([0 1 -120 0]);
grid on;
ylabel('dB')
snr=calculateSNR(spec(1:fB),f);
s=sprintf('SNR = %4.1fdB\n',snr)
text(0.5,-90,s);
s=sprintf('NBW=%7.5f',1.5/N);
text(0.5, -110, s);
```

simulateSNR

Synopsis: `[snr,amp] = simulateSNR(ntf,OSR,amp,f0=0,nlev=2,f=1/(4*OSR),k=13)`
Simulate a delta-sigma modulator with sine wave inputs of various amplitudes and calculate the signal-to-noise ratio (SNR) in dB for each input.

Arguments

ntf	The modulator NTF, given in zero-pole form.
OSR	The oversampling ratio. *OSR* is used to define the “band of interest.”
amp	A row vector listing the amplitudes to use. Defaults to [-120 -110...-20 -15 -10 -9 -8 … 0] dB, where 0 dB means a full-scale (peak value = *nlev*-1) sine wave.
f0	The center frequency of the modulator. $f0 \neq 0$ corresponds to a bandpass modulator.
nlev	The number of levels in the quantizer.
f	The normalized frequency of the test sinusoid; a check is made that the test frequency is in the band of interest. The frequency is adjusted so that it lies precisely in an FFT bin.
k	The number of time points used for the FFT is 2^k.

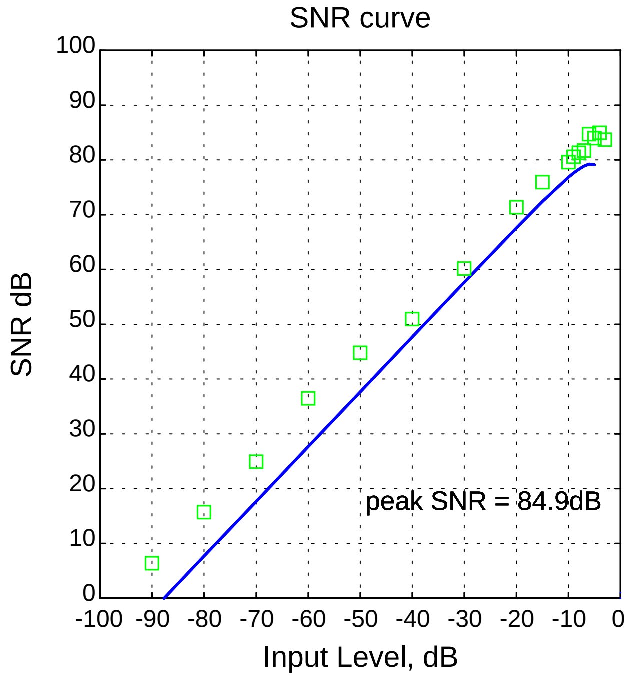
Output

snr	A row vector containing the SNR values calculated from the simulations.
amp	A row vector listing the amplitudes used.

Example

Compare the SNR vs. input amplitude curve for a fifth-order modulator determined by the describing function method with that determined by simulation.

```
>> OSR = 32; H = synthesizeNTF(5,OSR,1);
>> [snr_pred,amp] = predictSNR(H,OSR);
>> [snr,amp] = simulateSNR(H,OSR);
```




```
plot(amp,snr_pred,'b',amp,snr,'gs');
grid on;
figureMagic([-100 0], 10, 1, ...
[0 100], 10, 1);
xlabel('Input Level, dB');
ylabel('SNR dB');
title('SNR curve');
s=sprintf('peak SNR = %4.1fdB\n',...
max(snr));
text(-49,15,s);
```

realizeNTF

Synopsis: `[a,g,b,c] = realizeNTF(ntf,form='CRFB',stf=1)`
Convert a noise transfer function (NTF) into a set of coefficients for a particular modulator topology.

Arguments

ntf	The modulator NTF, given in zero-pole form (i.e. a zpk object).
form	A string specifying the modulator topology. CRFB Cascade-of-resonators, feedback form. CRFF Cascade-of-resonators, feedforward form. CIFB Cascade-of-integrators, feedback form. CIFF Cascade-of-integrators, feedforward form. Structures are described in detail in “MODULATOR MODEL DETAILS” on page 20.
stf	The modulator STF, specified as a zpk object. Note that the poles of the STF must match those of the NTF in order to guarantee that the STF can be realized without the addition of extra state variables.

Output

a	Feedback/feedforward coefficients from/to the quantizer ($1 \times n$).
g	Resonator coefficients ($1 \times \lfloor n/2 \rfloor$).
b	Feed-in coefficients from the modulator input to each integrator ($1 \times n+1$).
c	Integrator inter-stage coefficients. ($1 \times n$. In unscaled modulators, *c* is all ones.)

Example

Determine the coefficients for a 5th-order modulator with the cascade-of-resonators structure, feedback (CRFB) form.

```
>> H = synthesizeNTF(5,32,1);
>> [a,g,b,c] = realizeNTF(H,'CRFB')
a = 0.0007    0.0084    0.0550    0.2443    0.5579
g = 0.0028    0.0079
b = 0.0007    0.0084    0.0550    0.2443    0.5579    1.0000
c = 1     1     1     1     1
```

stuffABCD

Synopsis: `ABCD = stuffABCD(a,g,b,c,form='CRFB')`
Calculate the ABCD matrix given the parameters of a specified modulator topology.

Arguments

a	Feedback/feedforward coefficients from/to the quantizer. $1 \times n$
g	Resonator coefficients. $1 \times \lfloor n/2 \rfloor$
b	Feed-in coefficients from the modulator input to each integrator. $1 \times n + 1$
c	Integrator inter-stage coefficients. $1 \times n$
form	see `realizeNTF` on page 9 for a list of supported structures.

Output

ABCD	A state-space description of the modulator loop filter.

mapABCD

`[a,g,b,c] = mapABCD(ABCD,form='CRFB')`
Calculate the parameters for a specified modulator topology, assuming ABCD fits that topology.

Arguments

ABCD	A state-space description of the modulator loop filter.
form	see `realizeNTF` on page 9 for a list of supported structures.

Output

a	Feedback/feedforward coefficients from/to the quantizer. $1 \times n$
g	Resonator coefficients. $1 \times \lfloor n/2 \rfloor$
b	Feed-in coefficients from the modulator input to each integrator. $1 \times n + 1$
c	Integrator inter-stage coefficients. $1 \times n$

scaleABCD

Synopsis: `[ABCDs,umax]=scaleABCD(ABCD,nlev=2,f=0,xlim=1,ymax=nlev+5,umax,N=1e5)`
Scale the ABCD matrix so that the state maxima are less than a specified limit. The maximum stable input is determined as a side-effect of this process.

Arguments

ABCD	A state-space description of the modulator loop filter.
nlev	The number of levels in the quantizer.
f	The normalized frequency of the test sinusoid.
xlim	The limit on the states. May be given as a vector.
ymax	The threshold for judging modulator stability. If the quantizer input exceeds *ymax*, the modulator is considered to be unstable.

Output

*ABCD*s	The scaled state-space description of the modulator loop filter.
umax	The maximum stable input. Input sinusoids with amplitudes below this value should not cause the modulator states to exceed their specified limits.

calculateTF

Synopsis: `[ntf,stf] = calculateTF(ABCD,k=1)`
Calculate the NTF and STF of a delta-sigma modulator.

Arguments

ABCD	A state-space description of the modulator loop filter.
k	The value to use for the quantizer gain.

Output

ntf	The modulator NTF, given as an LTI system in zero-pole form.
stf	The modulator STF, given as an LTI system in zero-pole form.

Example

Realize a fifth-order modulator with the cascade-of-resonators structure, feedback form. Calculate the ABCD matrix of the loop filter and verify that the NTF and STF are correct.

```
>> H = synthesizeNTF(5,32,1)

Zero/pole/gain:
       (z-1) (z^2 - 1.997z + 1) (z^2 - 1.992z + 1)
----------------------------------------------------------
(z-0.7778) (z^2 - 1.613z + 0.6649) (z^2 - 1.796z + 0.8549)

Sampling time: 1
>> [a,g,b,c] = realizeNTF(H)
a =
    0.0007    0.0084    0.0550    0.2443    0.5579
g =
    0.0028    0.0079
b =
    0.0007    0.0084    0.0550    0.2443    0.5579    1.0000
c =
     1     1     1     1     1
>> ABCD = stuffABCD(a,g,b,c)
ABCD =
    1.0000         0         0         0         0    0.0007   -0.0007
    1.0000    1.0000   -0.0028         0         0    0.0084   -0.0084
    1.0000    1.0000    0.9972         0         0    0.0633   -0.0633
         0         0    1.0000    1.0000   -0.0079    0.2443   -0.2443
         0         0    1.0000    1.0000    0.9921    0.8023   -0.8023
         0         0         0         0    1.0000    1.0000         0
>> [ntf,stf] = calculateTF(ABCD)
Zero/pole/gain:
       (z-1) (z^2 - 1.997z + 1) (z^2 - 1.992z + 1)
----------------------------------------------------------
(z-0.7778) (z^2 - 1.613z + 0.6649) (z^2 - 1.796z + 0.8549)

Sampling time: 1

Zero/pole/gain:
1

Static gain.
```

simulateESL

Synopsis: `[sv,sx,sigma_se,max_sx,max_sy]`
`= simulateESL(v,mtf,M=16,dw=[1…], sx0=[0…])`

Simulate the element selection logic (ESL) of a multi-element DAC using a particular mismatch-shaping transfer function (`mtf`).

[1] R. Schreier and B. Zhang "Noise-shaped multibit D/A convertor employing unit elements," *Electronics Letters*, vol. 31, no. 20, pp. 1712-1713, Sept. 28 1995.

Arguments

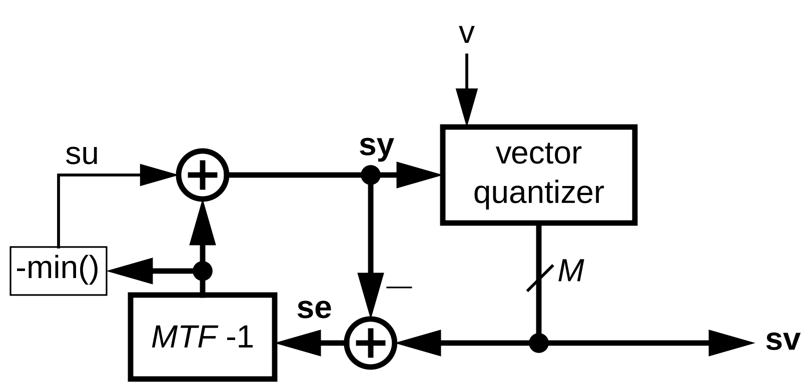
v	A vector containing the number of elements to enable. Note that the output of `simulateDSM` must be offset and scaled in order to be used here as *v* must be in the range $[0, \sum_i^M dw(i)]$.
mtf	The mismatch-shaping transfer function, given in zero-pole form.
M	The number of elements.
dw	A vector containing the weight associated with each element.
sx0	An $n \times M$ matrix containing the initial state of the element selection logic.

Output

sv	The selection vector: a vector of zeros and ones indicating which elements to enable.
sx	An $n \times M$ matrix containing the final state of the element selection logic.
sigma_se	The rms value of the selection error, $\boldsymbol{se} = \boldsymbol{sv} - \boldsymbol{sy}$. *sigma_se* may be used to analytically estimate the power of in-band noise caused by element mismatch.
max_sx	The maximum value attained by any state in the ESL.
max_sy	The maximum value attained by any component of the (un-normalized) "desired usage" vector.

Example

Run `dsdemo6.m`.



Block diagram of the Element Selection Logic

designHBF

Synopsis: `[f1,f2,info]=designHBF(fp=0.2,delta=1e-5,debug=0)`
Design a hardware-efficient linear-phase half-band filter for use in the decimation or interpolation filter associated with a delta-sigma modulator. This function is based on the procedure described by Saramäki [1]. Note that since the algorithm uses a non-deterministic search procedure, successive calls may yield different designs.

[1] T. Saramäki, "Design of FIR filters as a tapped cascaded interconnection of identical subfilters," *IEEE Transactions on Circuits and Systems*, vol. 34, pp. 1011-1029, 1987.

Arguments

fp	Normalized passband cutoff frequency.
delta	Passband and stopband ripple in absolute value.

Output

f1,f2 Prototype filter and subfilter coefficients and their canonical-signed digit (csd) representation.

info A vector containing the following information data (only set when `debug`=1):

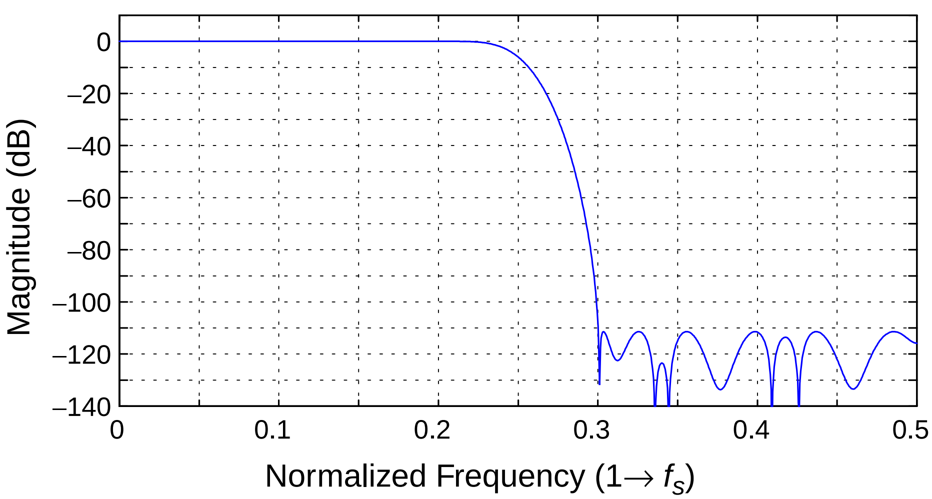
complexity	The number of additions per output sample.
n1,n2	The length of the f1 and f2 vectors.
sbr	The achieved stop-band attenuation (dB).
phi	The scaling factor for the F2 filter.

Example

Design of a lowpass half-band filter with a cut-off frequency of $0.2f_s$, a passband ripple of less than 10^{-5} and a stopband rejection of at least 10^{-5} (-100 dB).

```
>> [f1,f2] = designHBF(0.2,1e-5);
>> f = linspace(0,0.5,1024);
>> plot(f, dbv(frespHBF(f,f1,f2)))
```

A plot of the filter response is shown below. The filter achieves 109 dB of attenuation in the stopband and uses only 124 additions (no true multiplications) to produce each output sample.



The structure of this filter as a decimation or interpolation filter is shown below. The coefficients and their signed-digit decompositions are

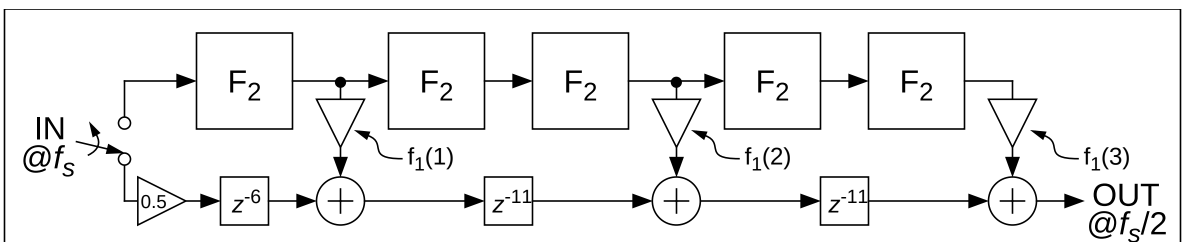
```
[f1.val]' =
    0.9453
   -0.6406
    0.1953
```

```
[f2.val]' =
    0.6211
   -0.1895
    0.0957
   -0.0508
    0.0269
   -0.0142
```

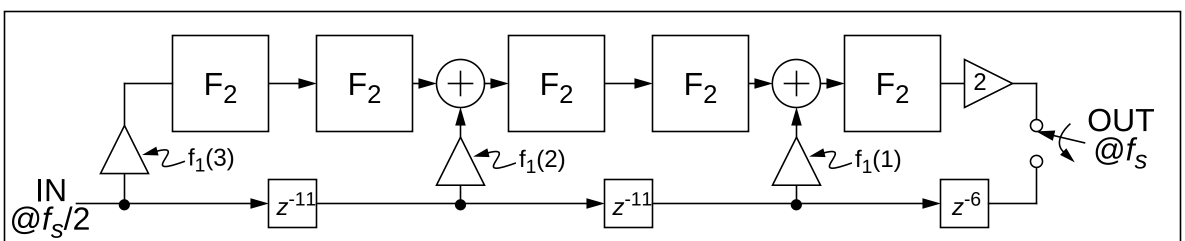
```
>> f1.csd
ans =
     0    -4    -7
     1    -1     1
ans =
    -1    -3    -6
    -1    -1    -1
ans =
    -2    -4    -7
     1    -1     1
```

```
>> f2.csd
ans =
    -1    -3    -8
     1     1    -1
ans =
    -2    -4    -9
    -1     1    -1
ans =
    -3    -5    -9
     1    -1     1
ans =
    -4    -7    -8
    -1     1     1
ans =
    -5    -8   -11
     1    -1    -1
ans =
    -6    -9   -11
    -1     1    -1
```

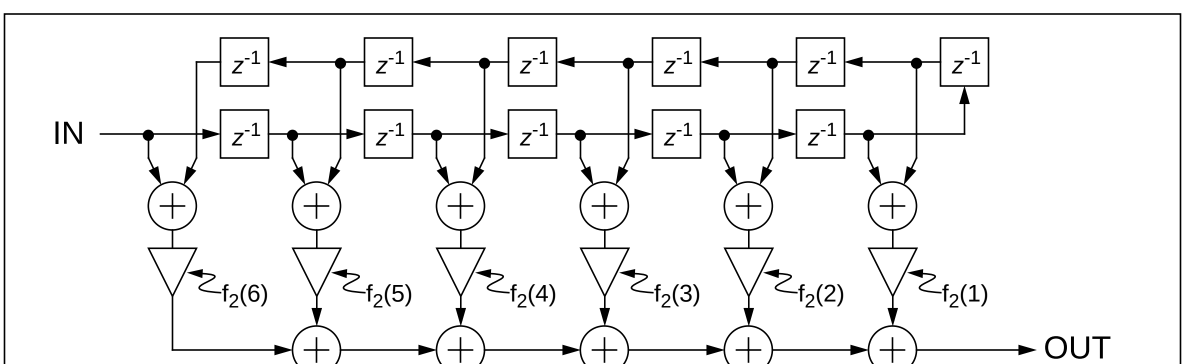
In the signed-digit expansions, the first row contain the powers of two while the second row gives their signs. For example, $f_1(1) = 0.9453 = 2^0 - 2^{-4} + 2^{-7}$ and $f_2(1) = 0.6211 = 2^{-1} + 2^{-3} - 2^{-8}$. Since the filter coefficients for this example use only 3 signed digits, each multiply-accumulate operation shown in the diagram below needs only 3 binary additions. Thus, an implementation of this 110th-order FIR filter needs to perform only $3\times3 + 5\times(3\times6 + 6 - 1) = 124$ additions at the low ($f_s/2$) rate.



Decimation structure.



Interpolation structure.



F_2 filter.

simulateHBF

Synopsis: `y = simulateHBF(x,f1,f2,mode=0)`
Simulate a Saramaki half-band filter (see `designHBF` on page 14) in the time domain.

Arguments

x	The input data.
f1,f2	Filter coefficients. f1 and f2 can be vectors of values or struct arrays like those returned from `designHBF`.
mode	The mode flag determines whether the input is filtered, interpolated, or decimated according to the following: 0 Plain filtering, no interpolation or decimation. 1 The input is interpolated 2 The output is decimated, even samples are taken. 3 The output is decimated, odd samples are taken.

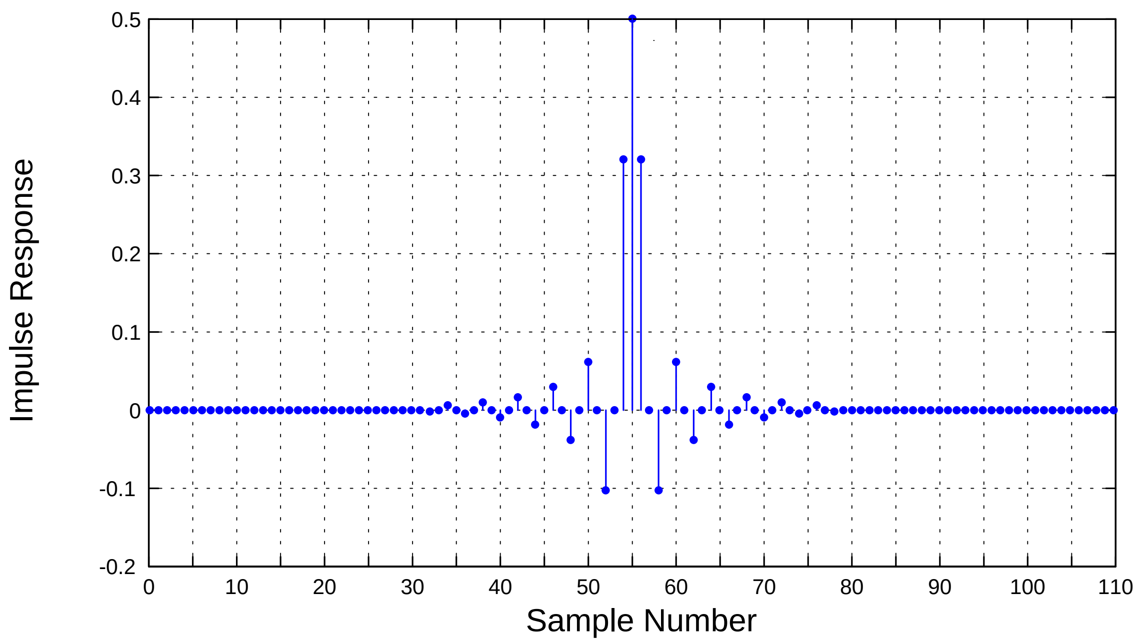
Output

y	The output data.

Example

Plot the impulse response of the HBF designed on the previous page.

```
>> N = (2*length(f1)-1)*2*(2*length(f2)-1)+1;
>> y = simulateHBF([1 zeros(1,N-1)],f1,f2);
>> stem([0:N-1],y);
>> figureMagic([0 N-1],5,2, [-0.2 0.5],0.1,1)
>> printmif('HBFimp', [6 3], 'Helvetica8')
```



designLCBP

Synopsis: `[param,H,L0,ABCD,x]=`
`designLCBP(n=3,OSR=64,opt=2,Hinf=1.6,f0=1/4,t=[0 1],form='FB',x0,dbg)`

Design a continuous-time LC-bandpass modulator consisting of resistors and LC tanks driven by transconductors and current-source DACs. Dynamic range and impedance scaling are not applied.

Arguments

n	Number of LC tanks in the loop filter.
OSR	Oversampling ratio, $OSR = f_s/(2f_b)$
opt	A flag indicating whether optimized NTF zeros should be used or not.
H_inf	The maximum out-of-band gain of the NTF. See `synthesizeNTF` on page 4.
f0	Modulator center frequency, default = $f_s/4$.
t	Start and end times of the DAC feedback pulse. Note that $t_1 = 0$ implies the use of a comparator with zero delay– an impractical situation.
form	The specific form of the modulator. See the table and diagram below.
dbg	Set this to a non-zero value to observe the optimization process in action.

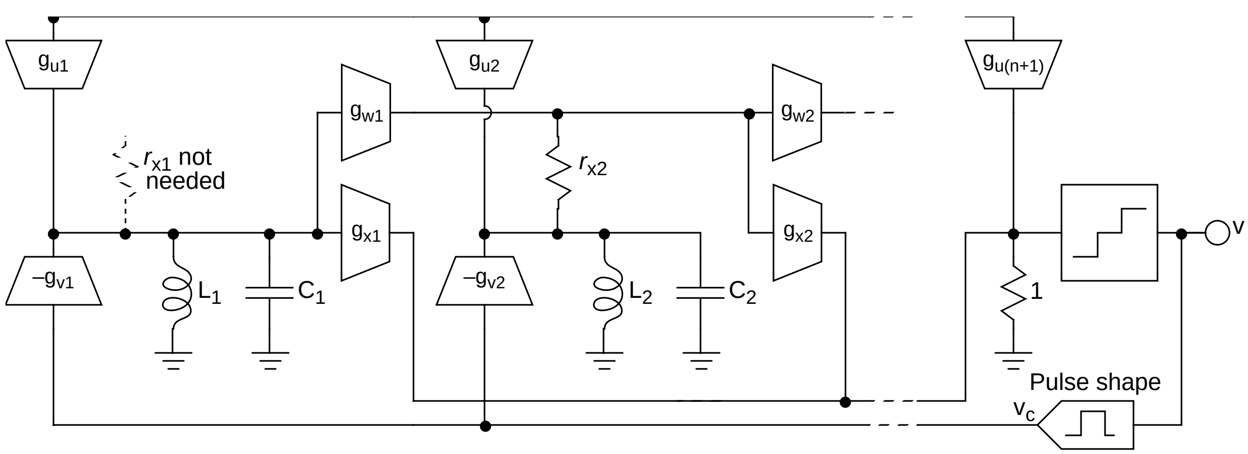
Output

param	A struct containing the (`n`, `OSR`, `Hinf`, `f0`, `t` and `form`) arguments plus the fields given in the table below.
H	The NTF of the equivalent discrete-time modulator.
L0	The continuous-time loop filter; an LTI object in zero-pole form.

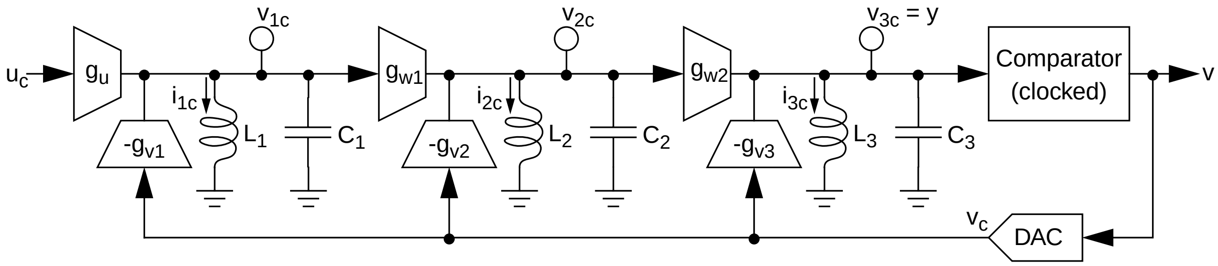
General LC topology and the coefficients subject to optimization for each of the supported forms:

<table>
<tr><th>Form</th><th>L</th><th>C</th><th>g_u: $1 \times (n+1)$</th><th>g_v: $1 \times n$</th><th>g_w: $1 \times (n-1)$</th><th>g_x: $1 \times n$</th><th>r_x: $1 \times n$</th></tr>
<tr><td>FB</td><td colspan="2" rowspan="3">$\frac{1}{\sqrt{2\pi f_0}}$</td><td>[1 0 …]*</td><td>$[x_1\ x_2 \ldots\ x_n]$</td><td>[1…]</td><td>[0 0… 1]</td><td>[0 …]</td></tr>
<tr><td>FF</td><td>[1 0 … 1]*</td><td>[1 0 …]</td><td>[1…]</td><td>$[x_1\ x_2 \ldots\ x_n]$</td><td>[0 …]</td></tr>
<tr><td>R</td><td>[1 0 … 1]*</td><td>$[x_1\ 0 \ldots]$</td><td>[1…]</td><td>[0 0… 1]</td><td>$[0\ x_2 \ldots x_n]$</td></tr>
</table>

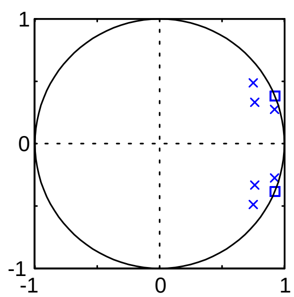
* scaled for unity STF gain at f_0.



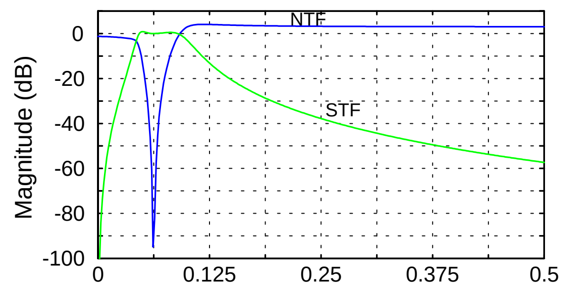
Example three-tank LC bandpass modulator (FB structure)



NTF Pole-Zero Plot



NTF and STF Frequency Response



Example

```
>> n = 3; OSR = 64; opt = 0; Hinf = 1.6; f0 = 1/16; t = [0.5 1]; form = 'FB';
>> [param,H,L0,ABCD,x] = designLCBP(n,OSR,opt,Hinf,f0,t,form);
>> plotPZ(H);
>> f = linspace(0,0.5,300); z = exp(2*pi*j*f);
>> ntf = dbv(evalTF(H,z)); stf = dbv(evalTFP(L0,H,f));
>> plot( f, ntf,'b', f, stf,'g');
>> figureMagic( [0,0.5],1/16,2, [-100 10],10,2);
gu = 1     0     0     0
gv = 0.653 2.703 3.708
gw = 1     1
gx = 0     0     1
rx = 0     0     0
```

See Also

`[H,L0,ABCD,k]=LCparam2tf(param,k=1)`

This function computes the transfer functions, quantizer gain and ABCD representation of an LC system. Inductor series resistance (`rl`) and capacitor shunt conductance (`gc`) can be incorporated into the calculations. Finite input and output conductance of the transconductors can be lumped with `gc`.

Bugs

The use of the `constr/fmincon` function (from the optimization toolbox, versions 5 & 6) makes convergence of `designLCBP` erratic and unreliable. In some cases, editing the `LCObj*` functions helps. A more robust optimizer/objective function, perhaps one which supports a step-size restriction, is needed.

`designLCBP` is outdated, now that LC modulators which use more versatile stages in the back-end have been developed. See [1] for an example design.

[1] R. Schreier, J. Lloyd, L. Singer, D. Paterson, M. Timko, M. Hensley, G. Patterson, K. Behel, J. Zhou and W. J, Martin, “A 10-300 MHz IF-digitizing IC with 90-105 dB dynamic range and 15-333 kHz bandwidth,” *IEEE Journal of Solid-State Circuits*, vol. SC-37, no. 12, pp. 1636-1644, Dec. 2002.

findPIS, find2dPIS (in the `PosInvSet` subdirectory)

Synopsis:
```
[s,e,n,o,Sc] = findPIS(u,ABCD,nlev=2,options)
options = [dbg=0 itnLimit=2000 expFactor=0.005 N=1000 skip=100
           qhullArgA=0.999 qhullArgC=.001]
s = find2dPIS(u,ABCD,options)
options = [dbg=0 itnLimit=100 expFactor=0.01 N=1000 skip=100]
```
Find a convex positively-invariant set for a delta-sigma modulator. `findPIS` requires compilation of the `qhull mex` file; `find2dPIS` does not, but is limited to second-order systems.

Arguments

u	The input to the modulator. If *u* is a scalar, the input to the modulator is constant. If *u* is a 2×1 vector, the input to the modulator may be any sequence whose samples lie in the range $[u(1), u(2)]$.
ABCD	A state-space description of the modulator loop filter.
nlev	The number of quantizer levels.
dbg	Set dbg=1 to get a graphical display of the iterations.
itnLimit	The maximum number of iterations.
expFactor	The expansion factor applied to the hull before every mapping operation. Increasing expFactor decreases the number of iterations but results in sets which are larger than they need to be.
N	The number of points to use when constructing the initial guess.
skip	The number of time steps to run the modulator before observing the state. This handles the possibility of “transients” in the modulator.
qhullArgA	The ‘A’ argument to the qhull program. Adjacent facets are merged if the cosine of the angle between their normals is greater than the absolute value of this parameter. Negative values imply that the merge operation is performed during hull construction, rather than as a post-processing step.
qhullArgC	The ‘C’ argument to the qhull program. A facet is merged into its neighbor if the distance between the facet’s centrum (the average of the facet’s vertices) and the neighboring hyperplane is less than the absolute value of this parameter. As with the above argument, negative values imply pre-merging while positive values imply post-merging.

Output

s	The vertices of the set ($dim \times n_v$)).
e	The edges of the set, listed as pairs of vertex indices ($2 \times n_e$).
n	The normals for the facets of the set ($dim \times n_f$).
o	The offsets for the facets of the set ($1 \times n_f$).
Sc	The scaling matrix which was used internally to “round out” the set.

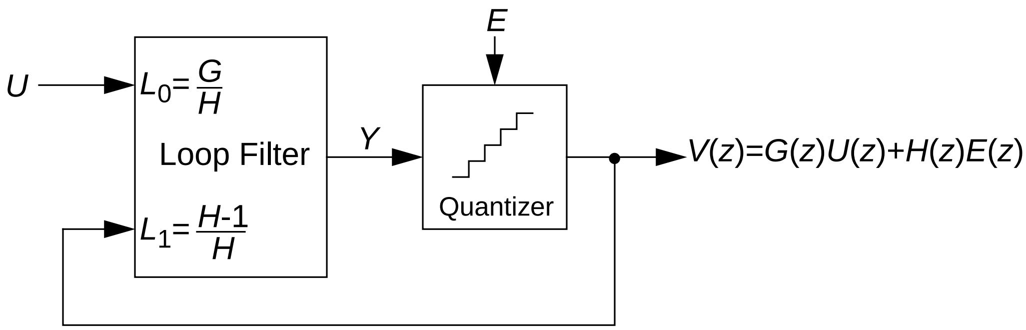
Background

This is an implementation of the method described in [1].

[1] R. Schreier, M. Goodson and B. Zhang “An algorithm for computing convex positively invariant sets for delta-sigma modulators,” *IEEE Transactions on Circuits and Systems I*, vol. 44, no. 1, pp. 38-44, January 1997.

MODULATOR MODEL DETAILS

A delta-sigma modulator with a single quantizer is assumed to consist of quantizer connected to a loop filter as shown in the diagram below.



The Loop Filter

The loop filter is described by an ABCD matrix. For single-quantizer systems, the loop filter is a two-input, one-output linear system and ABCD is an $(n+1)\times(n+2)$ matrix, partitioned into A $(n\times n)$, B $(n\times 2)$, C $(1\times n)$ and D (1×2) sub-matrices as shown below:

$$ABCD = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]. \tag{1}$$

The equations for updating the state and computing the output of the loop filter are

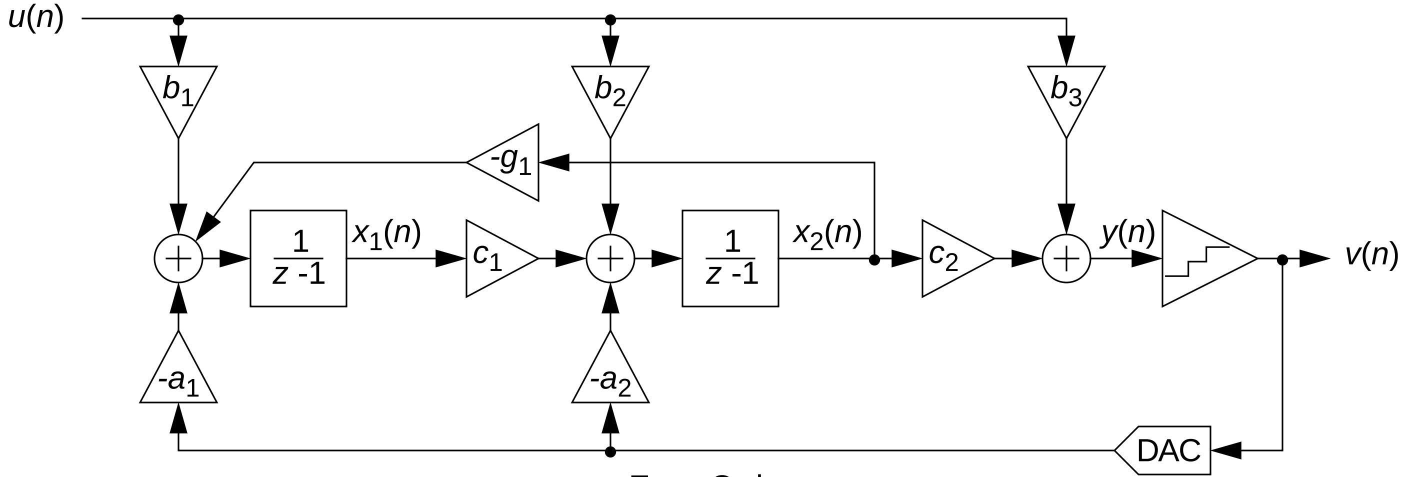
$$\begin{aligned} x(n+1) &= Ax(n) + B\begin{bmatrix} u(n) \\ v(n) \end{bmatrix} \\ y(n) &= Cx(n) + D\begin{bmatrix} u(n) \\ v(n) \end{bmatrix}. \end{aligned} \tag{2}$$

This formulation is sufficiently general to encompass all single-quantizer modulators which employ linear loop filters. The toolbox currently supports translation to/from an ABCD description and coefficients for the following topologies:

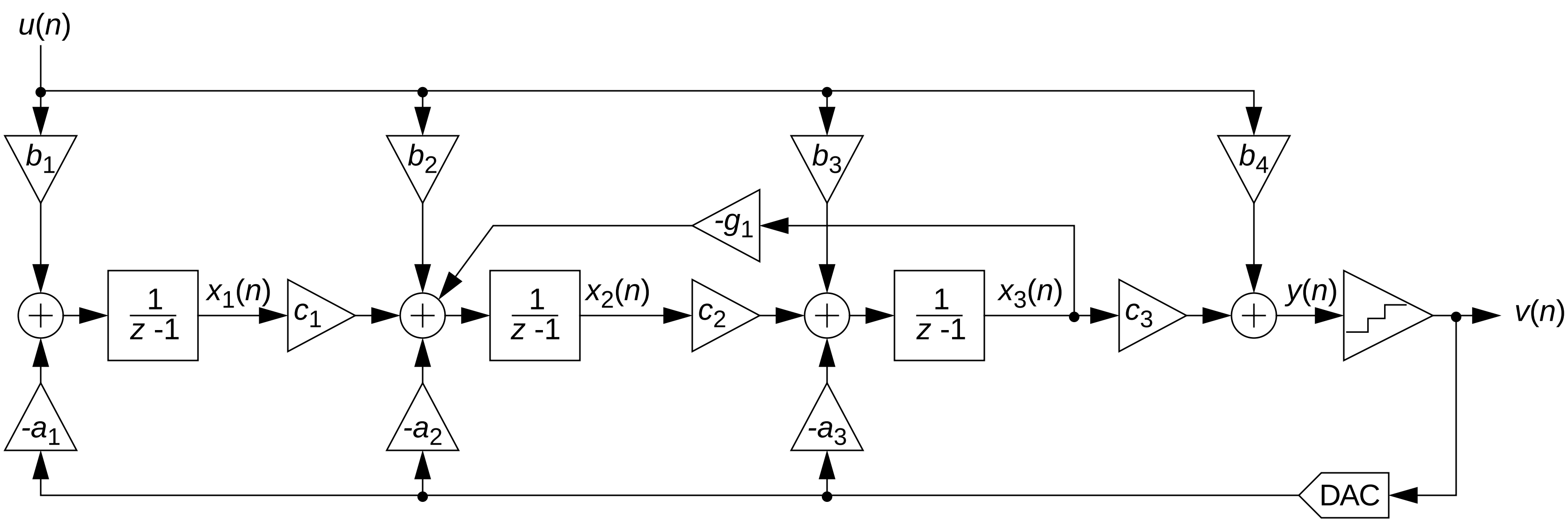
CIFB	Cascade-of-integrators, feedback form.
CIFF	Cascade-of-integrators, feedforward form.
CRFB	Cascade-of-resonators, feedback form.
CRFF	Cascade-of-resonators, feedforward form.

Multi-input and multi-quantizer systems are also described with an ABCD matrix and Eq. (2) still applies. For an n_i-input, n_o-output modulator, the dimensions of the sub-matrices are A: $n\times n$, B: $n\times(n_i+n_o)$, C: $n_o\times n$ and D: $n_o\times(n_i+n_o)$.

CIFB Structure

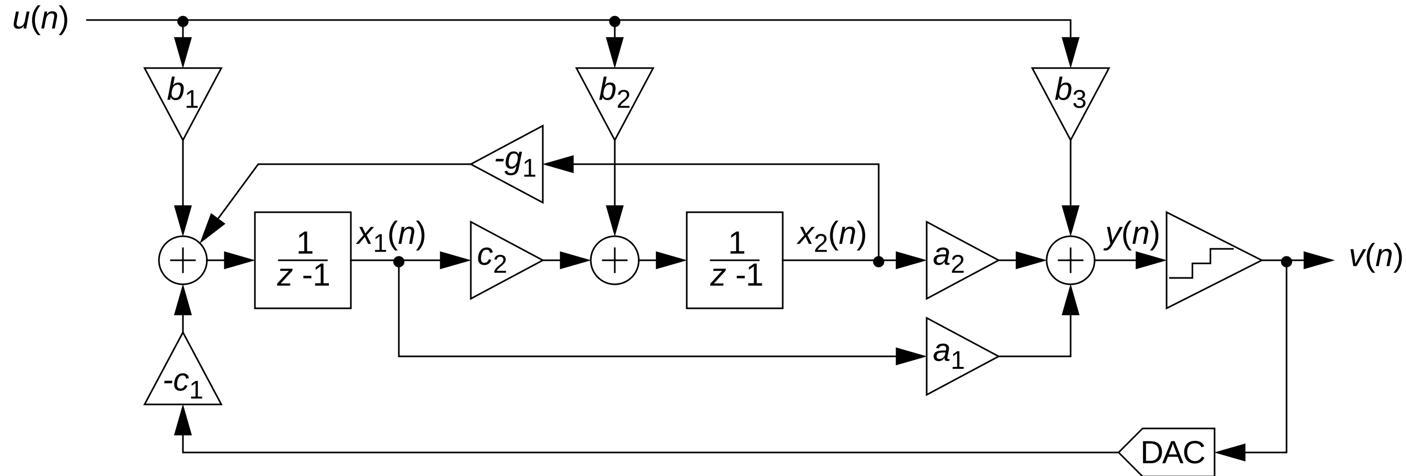


Even Order

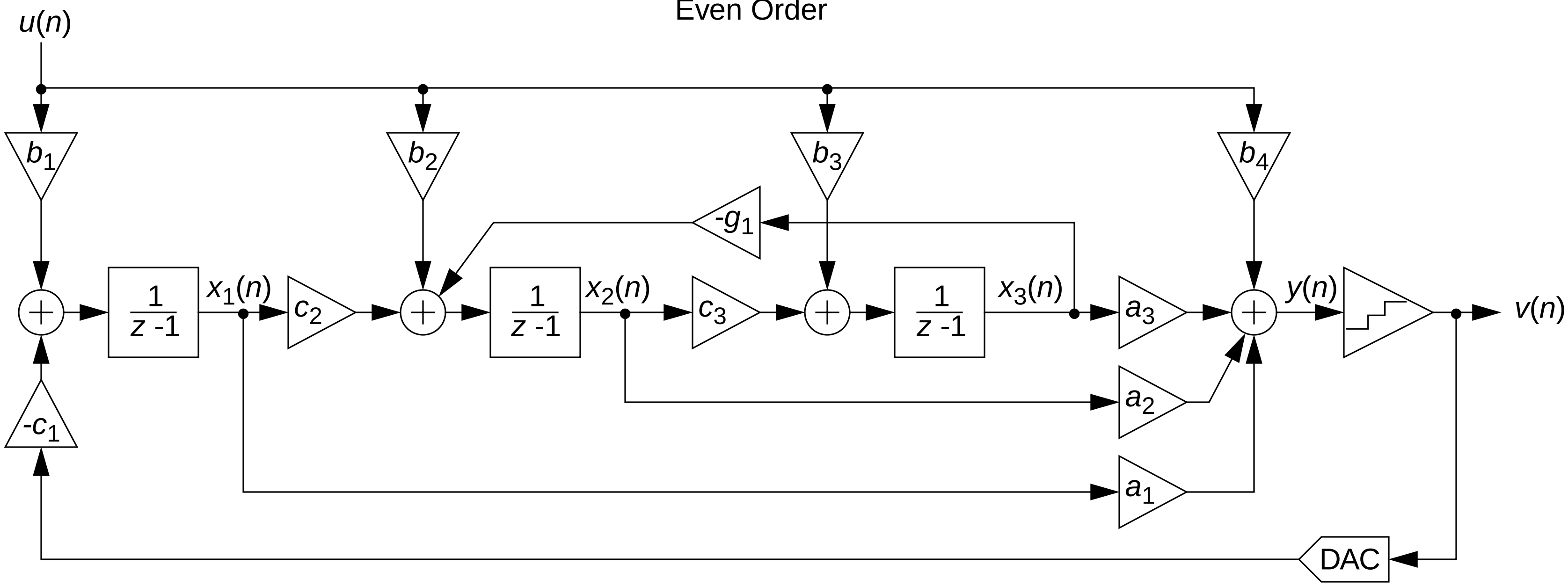


Odd Order

CIFF Structure

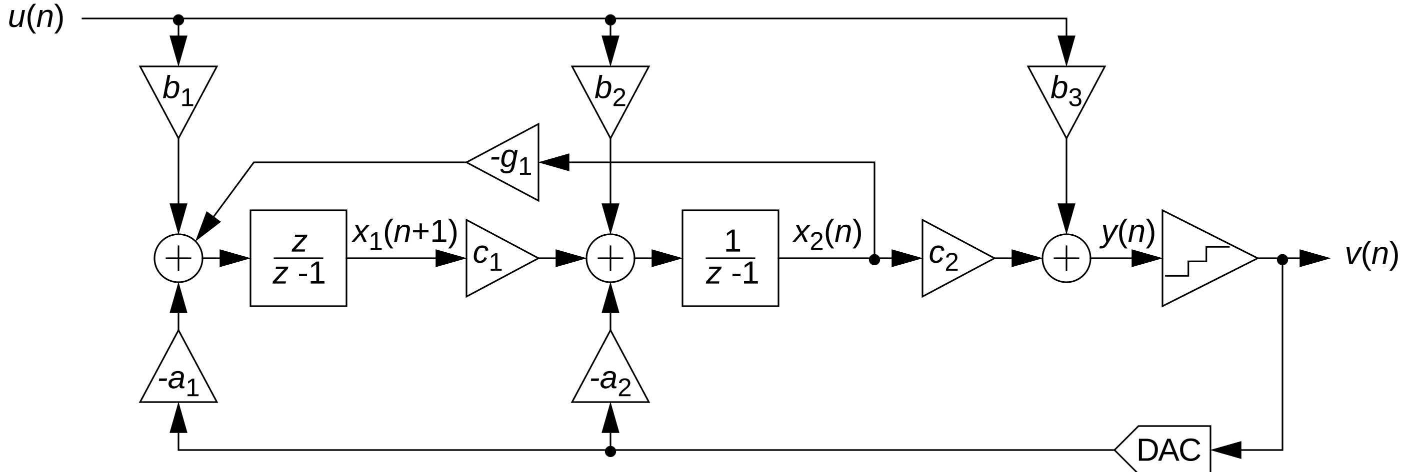


Even Order

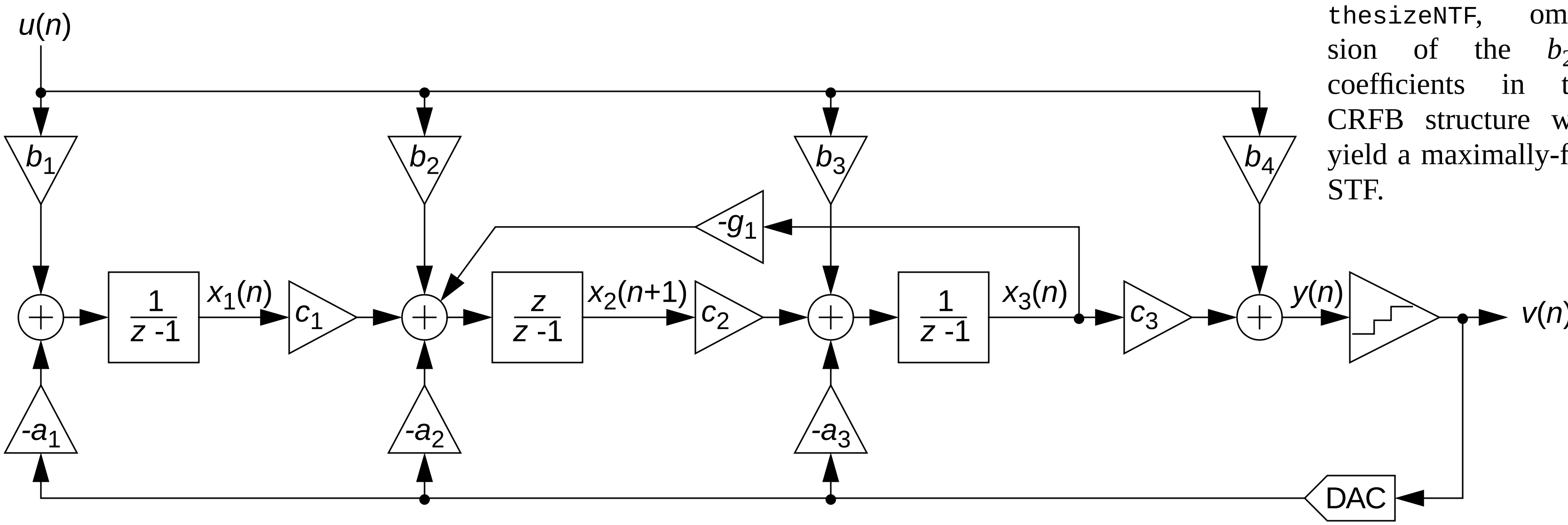


Odd Order

CRFB Structure



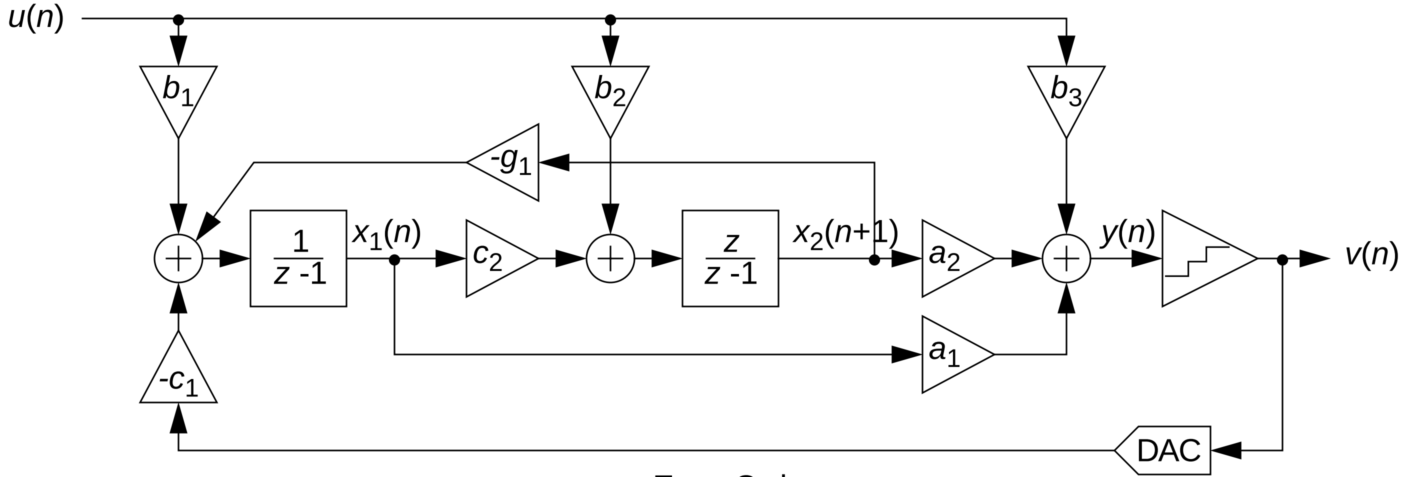
Even Order



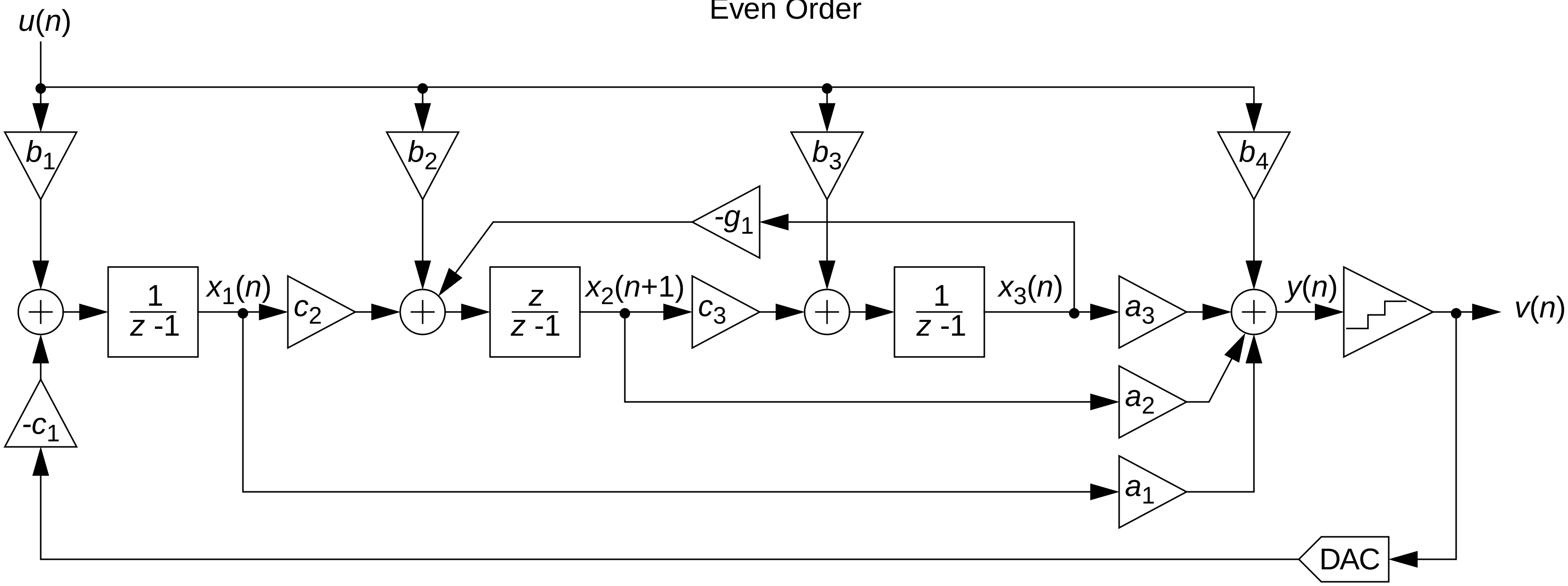
Odd Order

Note that with NTFs designed using `synthesizeNTF`, omission of the $b_{2\ldots}$ coefficients in the CRFB structure will yield a maximally-flat STF.

CRFF Structure



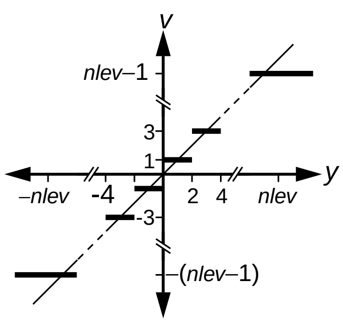
Even Order



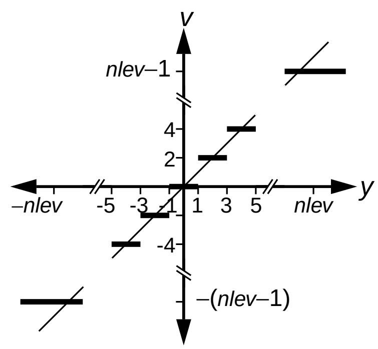
Odd Order

The Quantizer

The quantizer is ideal, producing integer outputs centered about zero. Quantizers with an even number of levels are of the mid-rise type and produce outputs which are odd integers. Quantizers with an odd number of levels are of the mid-tread type and produce outputs which are even integers.



Transfer curve of a quantizer with an even number of levels.



Transfer curve of a quantizer with an odd number of levels.