

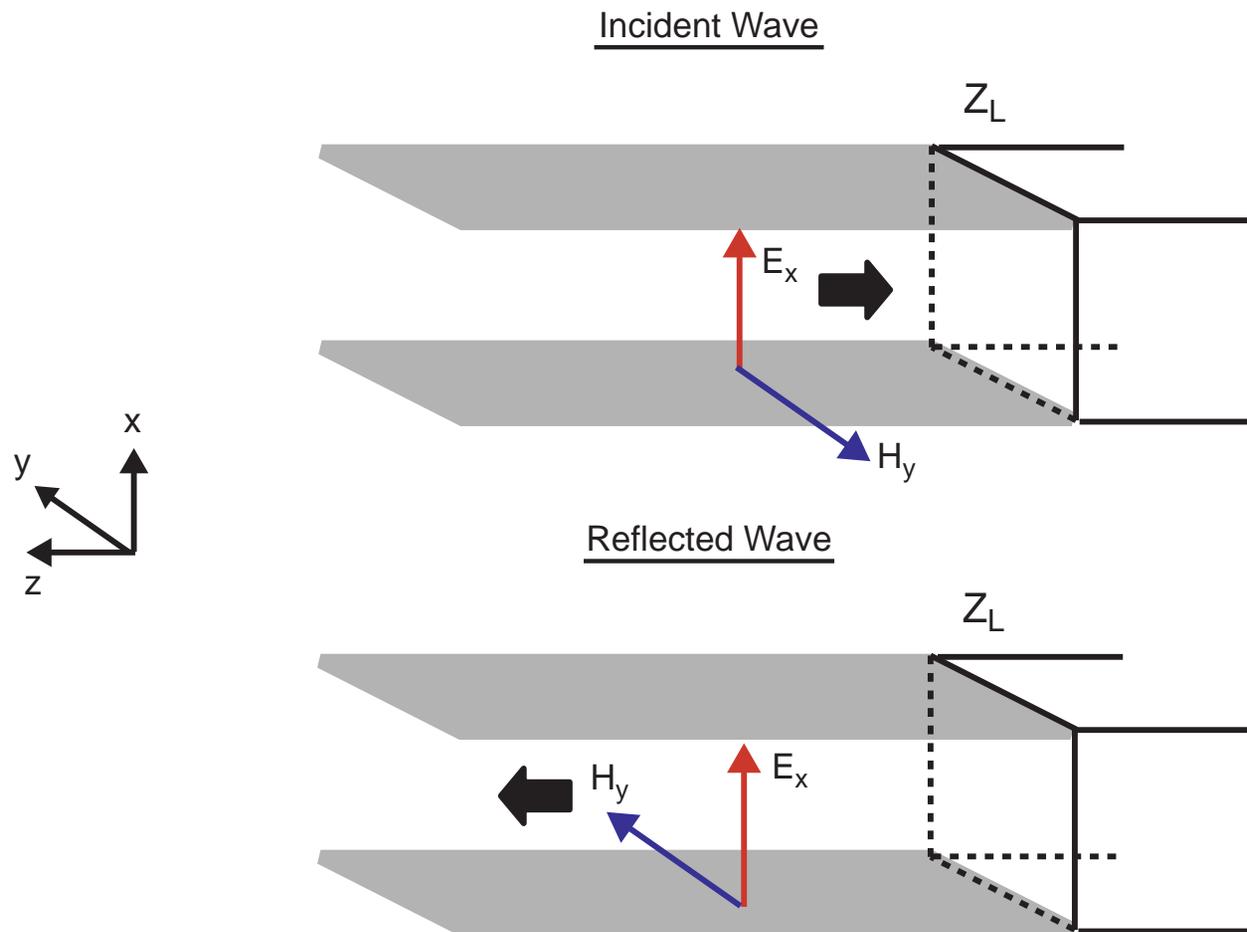
***High Speed Communication Circuits and Systems***  
***Lecture 3***  
***S-Parameters and Impedance Transformers***

**Michael H. Perrott**  
**February 9, 2004**

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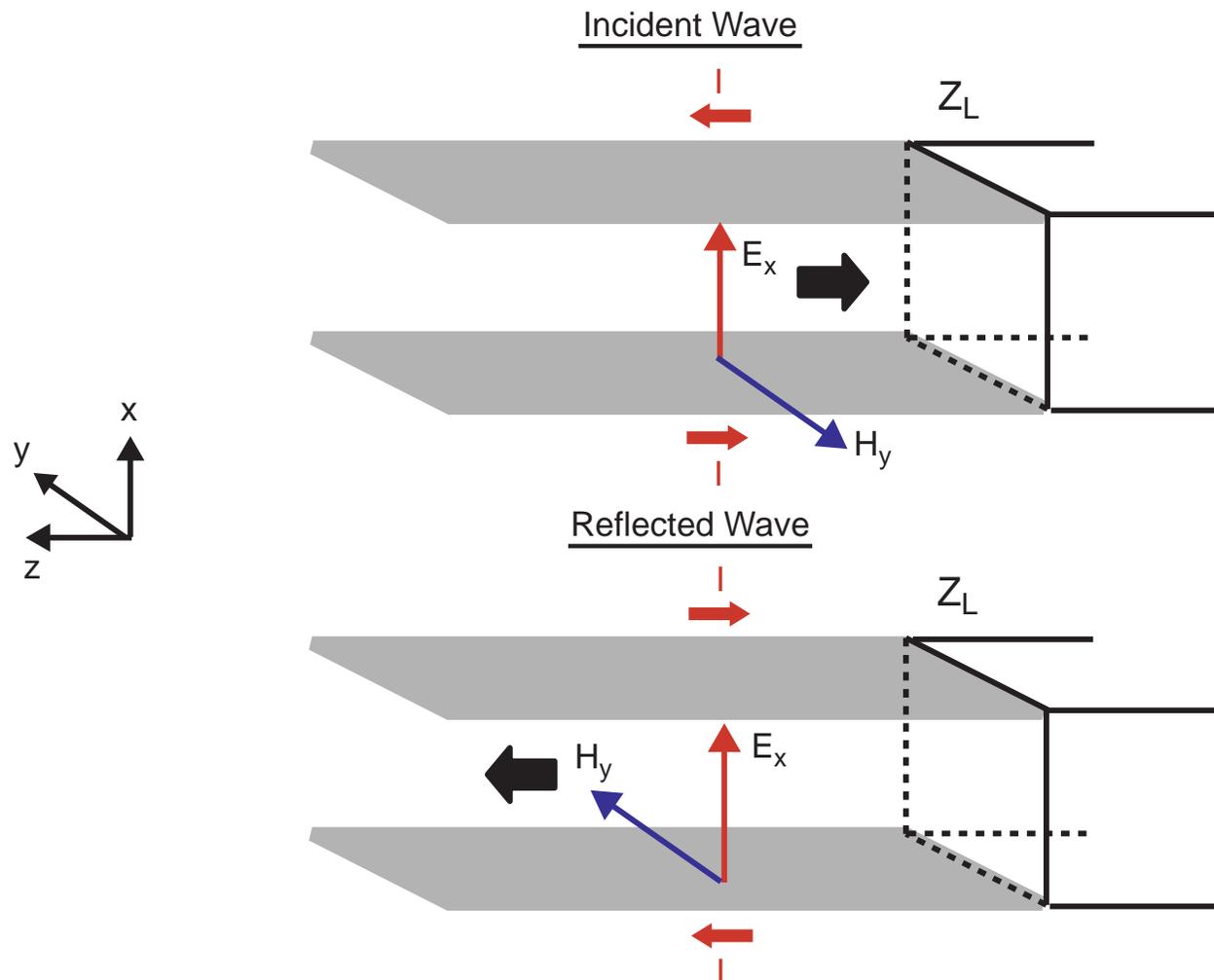
# What Happens When the Wave Hits a Boundary?

- Reflections can occur



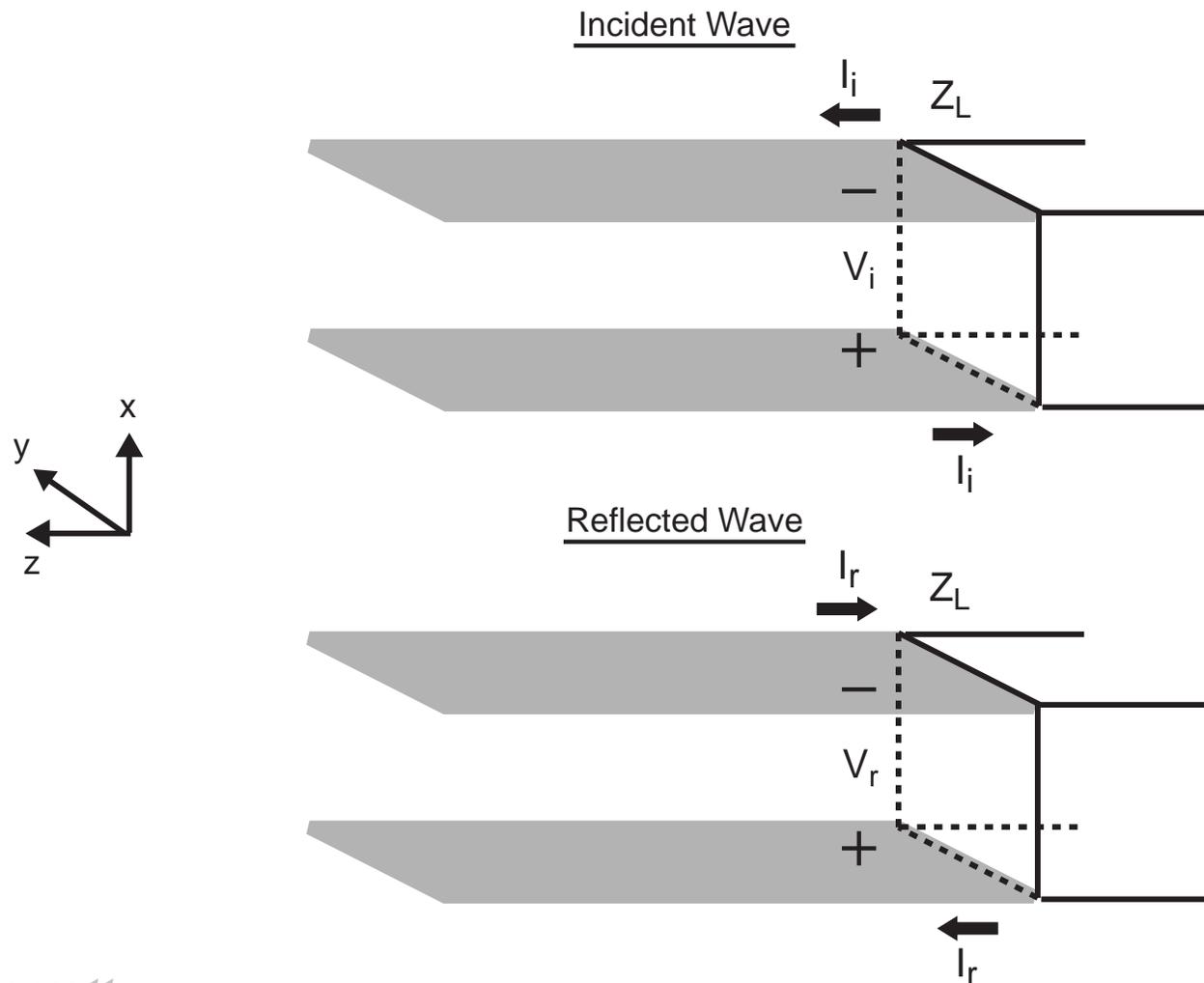
# What Happens When the Wave Hits a Boundary?

- At boundary
  - Orientation of H-field flips with respect to E-field
    - Current reverses direction with respect to voltage



# What Happens At The Load Location?

- Voltage and currents at load are ratioed according to the load impedance



## Voltage at Load

$$V_i + V_r$$

## Current at Load

$$I_i - I_r$$

## Ratio at Load

$$\frac{V_i + V_r}{I_i - I_r} = Z_L$$

## Relate to Characteristic Impedance

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- From previous slide

$$\frac{V_i + V_r}{I_i - I_r} = \frac{V_i}{I_i} \left( \frac{1 + V_r/V_i}{1 - I_r/I_i} \right) = Z_L$$

- Voltage and current ratio in transmission line set by its characteristic impedance

$$\frac{V_i}{I_i} = \frac{V_r}{I_r} = Z_0 \quad \Rightarrow \quad \frac{I_r}{I_i} = \frac{V_r}{V_i}$$

- Substituting:

$$Z_0 \left( \frac{1 + V_r/V_i}{1 - V_r/V_i} \right) = Z_L$$

## Define Reflection Coefficient

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- **Definition:**  $\Gamma_L = \frac{V_r}{V_i}$

- No reflection if  $\Gamma_L = 0$

- **Relation to load and characteristic impedances**

$$Z_o \left( \frac{1 + \Gamma_L}{1 - \Gamma_L} \right) = Z_L$$

- **Alternate expression**

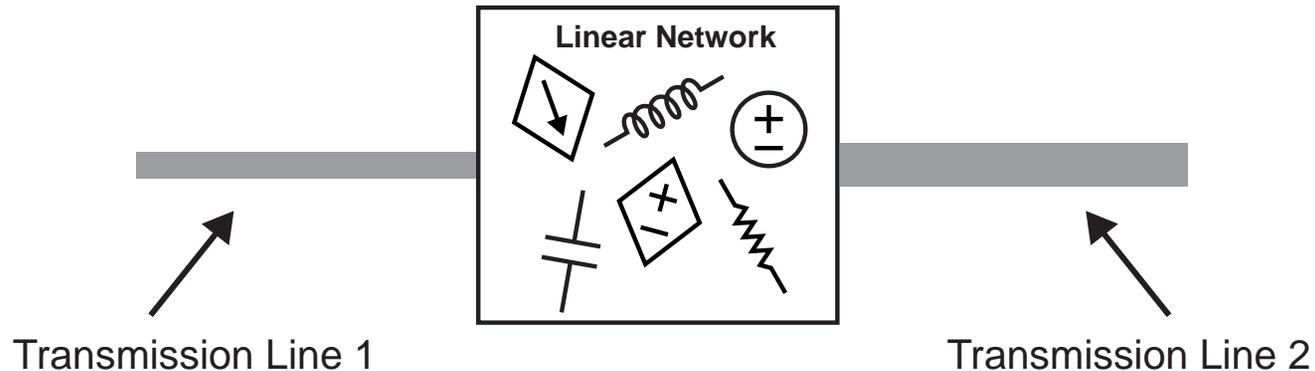
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

- No reflection if  $Z_L = Z_o$

# Parameterization of High Speed Circuits/Passives

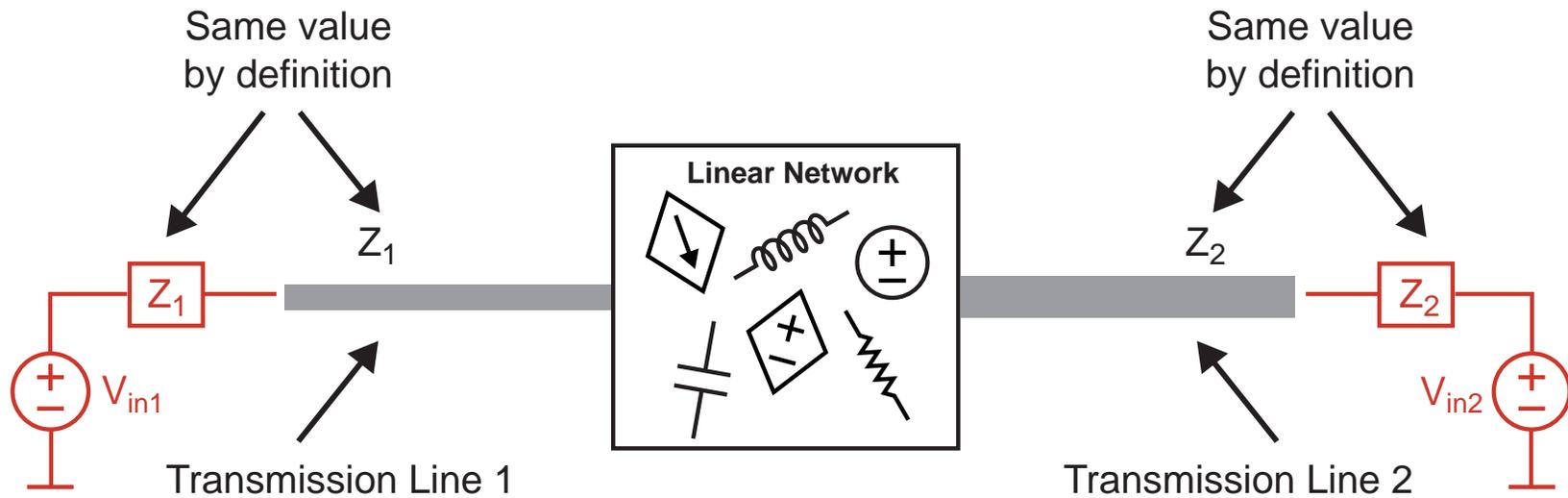
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- Circuits or passive structures are often connected to transmission lines at high frequencies
  - How do you describe their behavior?



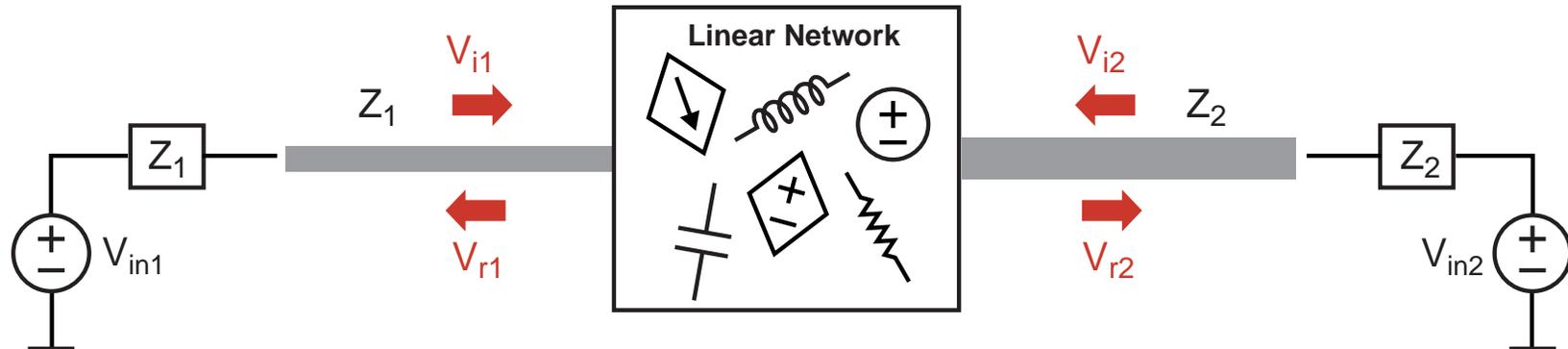
# Calculate Response to Input Voltage Sources

- Assume source impedances match their respective transmission lines



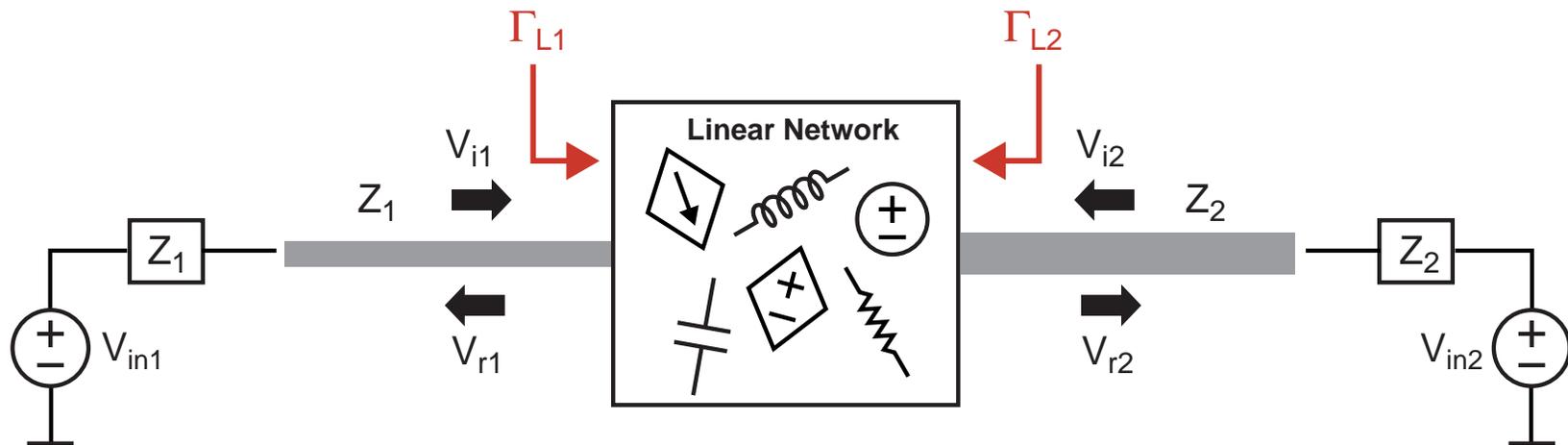
# Calculate Response to Input Voltage Sources

- Sources create incident waves on their respective transmission line
- Circuit/passive network causes
  - Reflections on same transmission line
  - Feedthrough to other transmission line



# Calculate Response to Input Voltage Sources

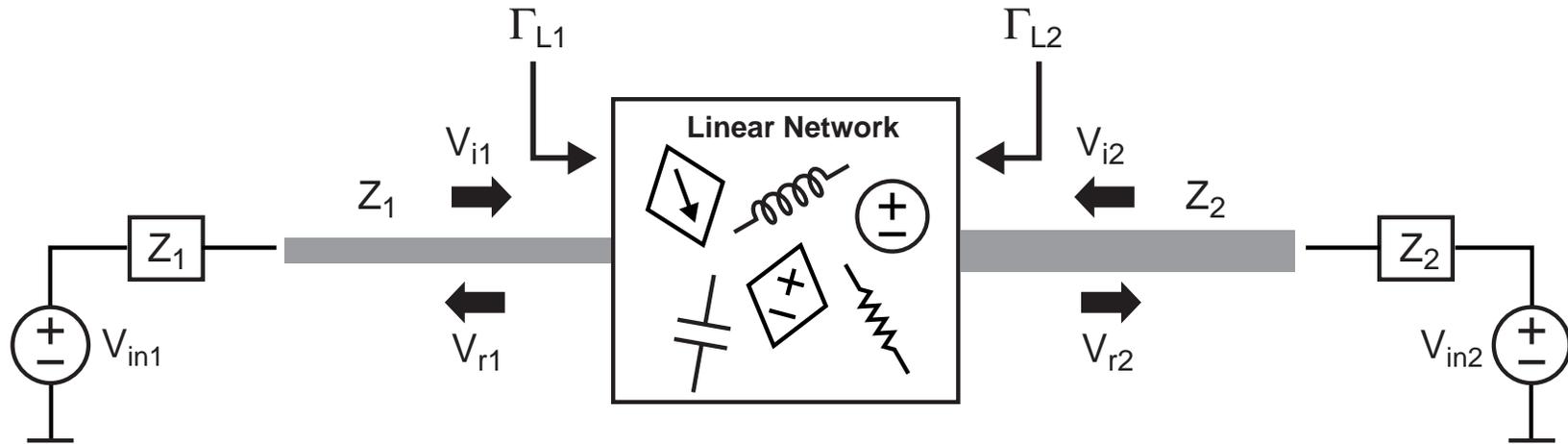
- Reflections on same transmission line are parameterized by  $\Gamma_L$ 
  - Note that  $\Gamma_L$  is generally different on each side of the circuit/passive network



How do we parameterize feedthrough to the other transmission line?

# S-Parameters – Definition

- Model circuit/passive network using 2-port techniques
  - Similar idea to Thevenin/Norton modeling

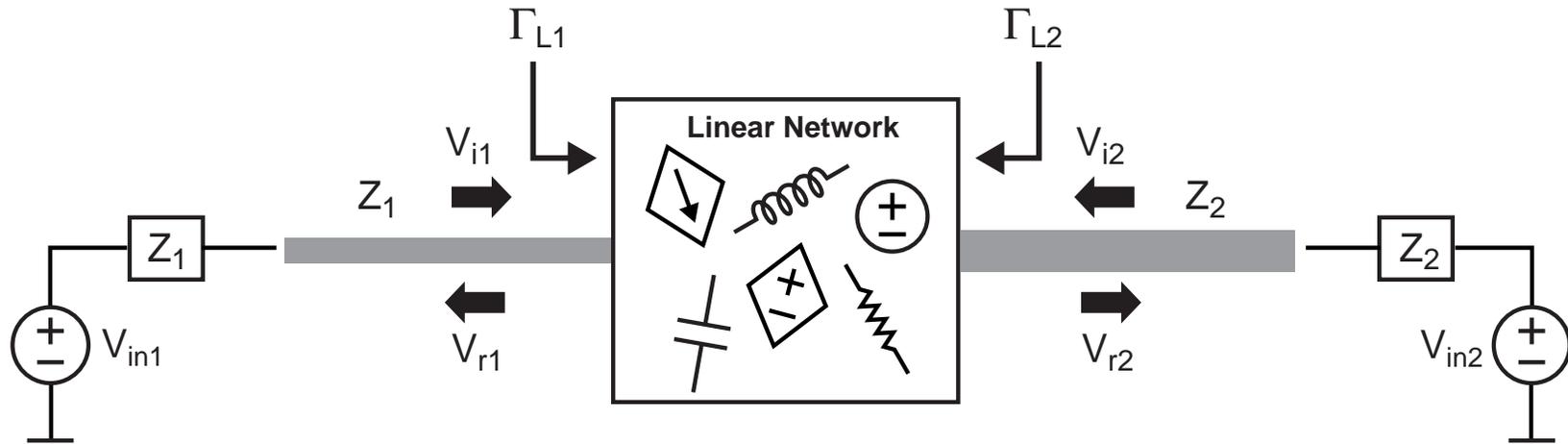


- Defining equations:

$$\frac{V_{r1}}{\sqrt{Z_1}} = S_{11} \frac{V_{i1}}{\sqrt{Z_1}} + S_{12} \frac{V_{i2}}{\sqrt{Z_2}}$$

$$\frac{V_{r2}}{\sqrt{Z_2}} = S_{21} \frac{V_{i1}}{\sqrt{Z_1}} + S_{22} \frac{V_{i2}}{\sqrt{Z_2}}$$

# S-Parameters – Calculation/Measurement



$$\frac{V_{r1}}{\sqrt{Z_1}} = S_{11} \frac{V_{i1}}{\sqrt{Z_1}} + S_{12} \frac{V_{i2}}{\sqrt{Z_2}}$$

$$\frac{V_{r2}}{\sqrt{Z_2}} = S_{21} \frac{V_{i1}}{\sqrt{Z_1}} + S_{22} \frac{V_{i2}}{\sqrt{Z_2}}$$

set  $V_{in2} = 0$

$$\Rightarrow S_{11} = \frac{V_{r1}}{V_{i1}} = \Gamma_{L1}$$

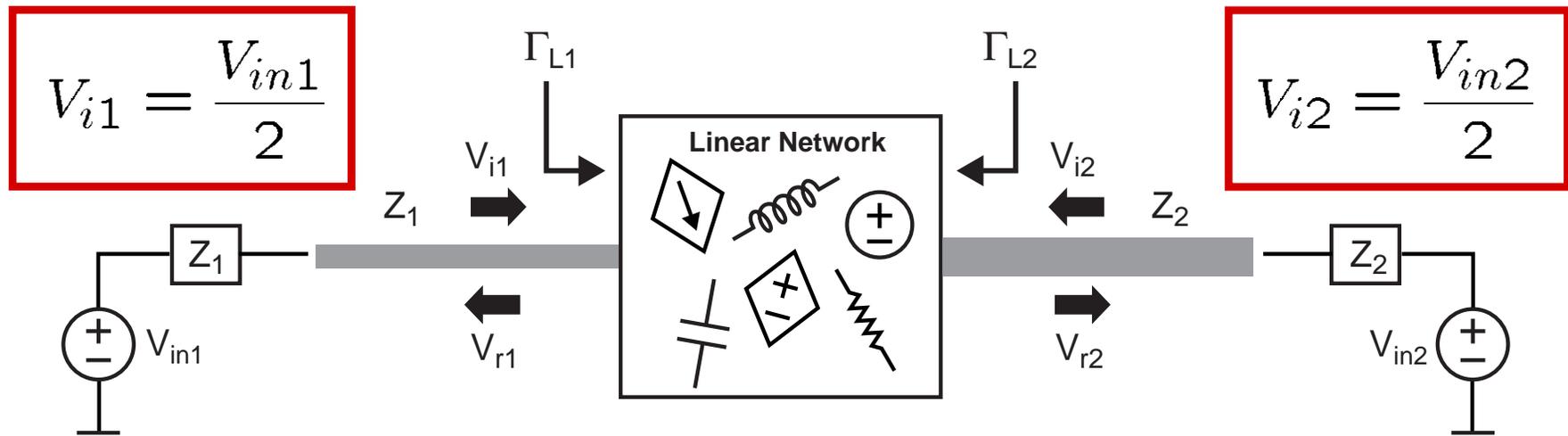
$$\Rightarrow S_{21} = \sqrt{\frac{Z_1}{Z_2}} \left( \frac{V_{r2}}{V_{i1}} \right)$$

set  $V_{in1} = 0$

$$\Rightarrow S_{22} = \frac{V_{r2}}{V_{i2}} = \Gamma_{L2}$$

$$\Rightarrow S_{12} = \sqrt{\frac{Z_2}{Z_1}} \left( \frac{V_{r1}}{V_{i2}} \right)$$

## Note: Alternate Form for $S_{21}$ and $S_{12}$



$$V_{i1} = \frac{V_{in1}}{2}$$

$$V_{i2} = \frac{V_{in2}}{2}$$

set  $V_{in2} = 0$

$$\Rightarrow S_{11} = \frac{V_{r1}}{V_{i1}} = \Gamma_{L1}$$

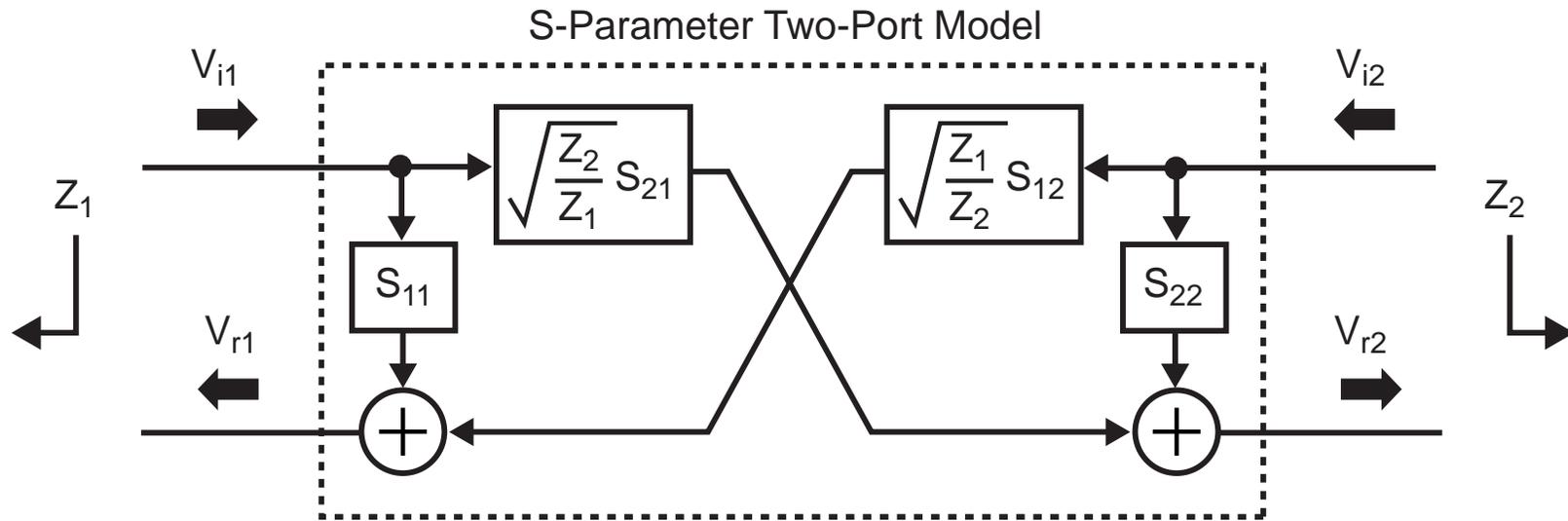
$$\Rightarrow S_{21} = 2\sqrt{\frac{Z_1}{Z_2}} \left( \frac{V_{r2}}{V_{in1}} \right)$$

set  $V_{in1} = 0$

$$\Rightarrow S_{22} = \frac{V_{r2}}{V_{i2}} = \Gamma_{L2}$$

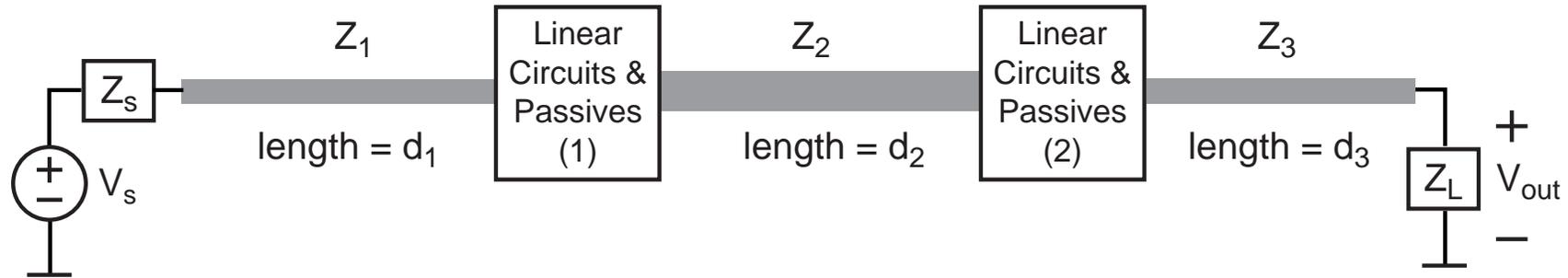
$$\Rightarrow S_{12} = 2\sqrt{\frac{Z_2}{Z_1}} \left( \frac{V_{r1}}{V_{in2}} \right)$$

# Block Diagram of S-Parameter 2-Port Model



- **Key issue – two-port is parameterized with respect to the left and right side load impedances ( $Z_1$  and  $Z_2$ )**
  - Need to recalculate  $S_{11}$ ,  $S_{21}$ , etc. if  $Z_1$  or  $Z_2$  changes
  - Typical assumption is that  $Z_1 = Z_2 = 50$  Ohms

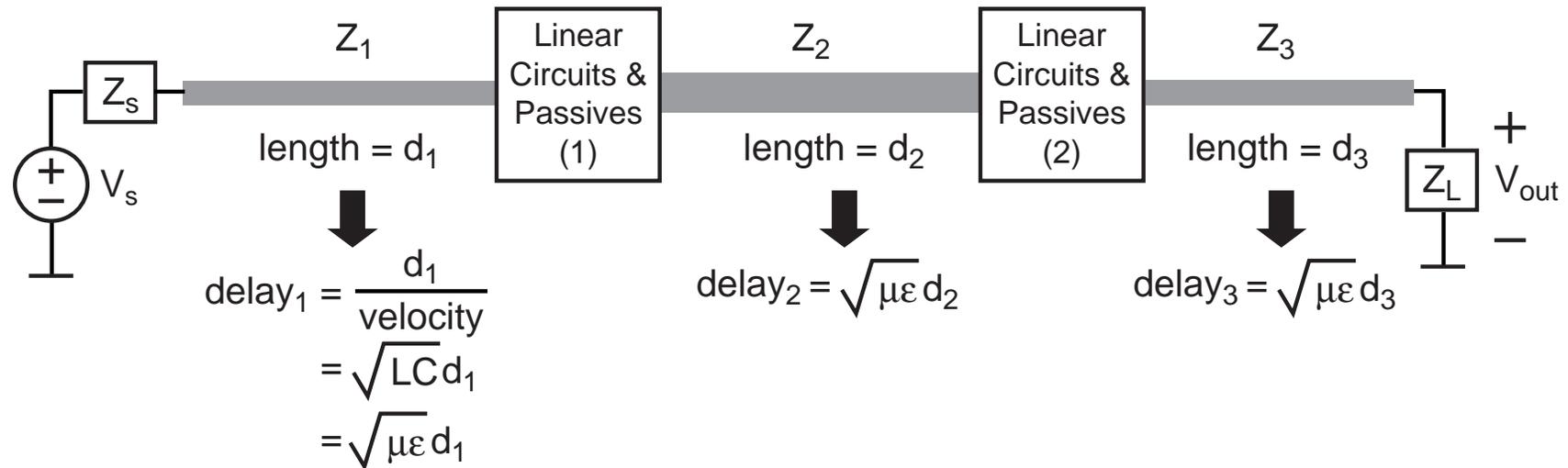
# Macro-modeling for Distributed, Linear Networks



- **Key parameters for a transmission line**
  - Characteristic impedance (only impacts S-parameter calculations)
  - Delay (function of length and  $\mu, \epsilon$ )
  - Loss (ignore for now)
- **Key parameters for circuits/passives**
  - S-parameters

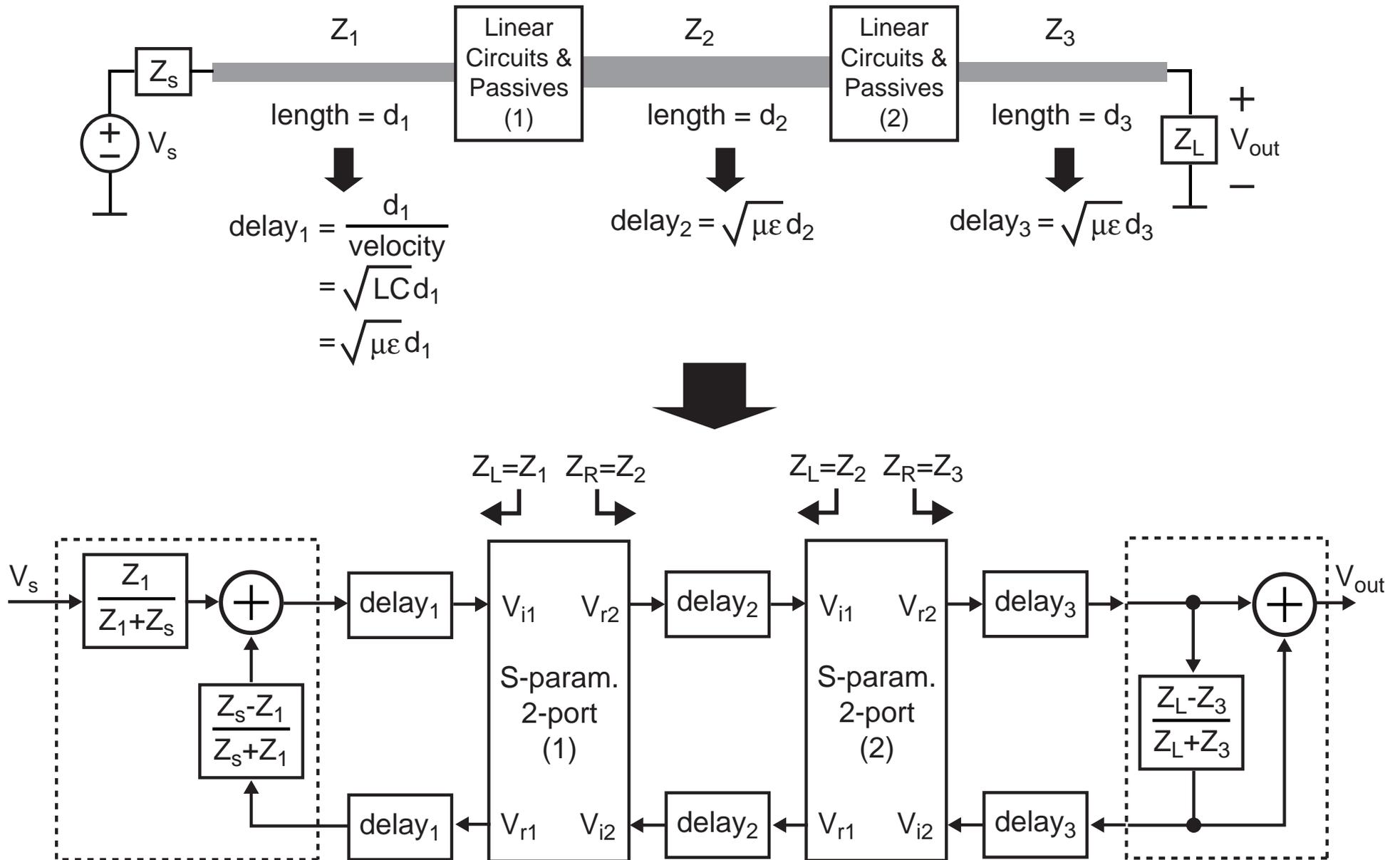
**We would like an overall macro-model for simulation**

# Macro-modeling for Distributed, Linear Networks

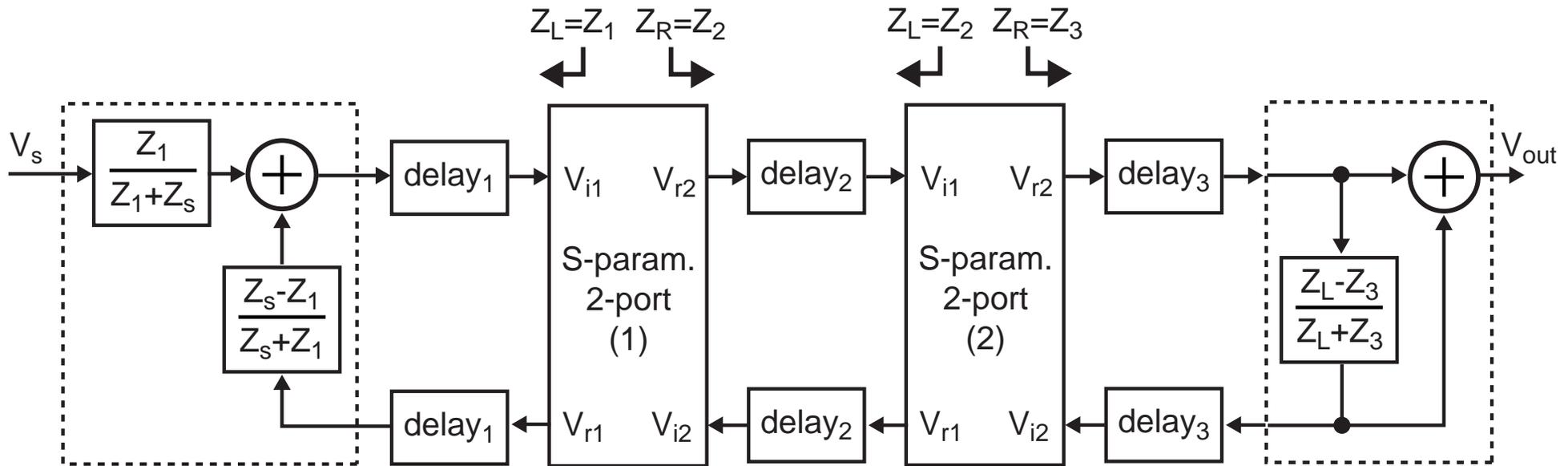


- **Model transmission line as a delay element**
  - If lossy, could also add an attenuation factor (which is a function of its length)
- **Model circuits/passives with S-parameter 2-ports**
- **Model source and load with custom blocks**

# Macro-modeling for Distributed, Linear Networks

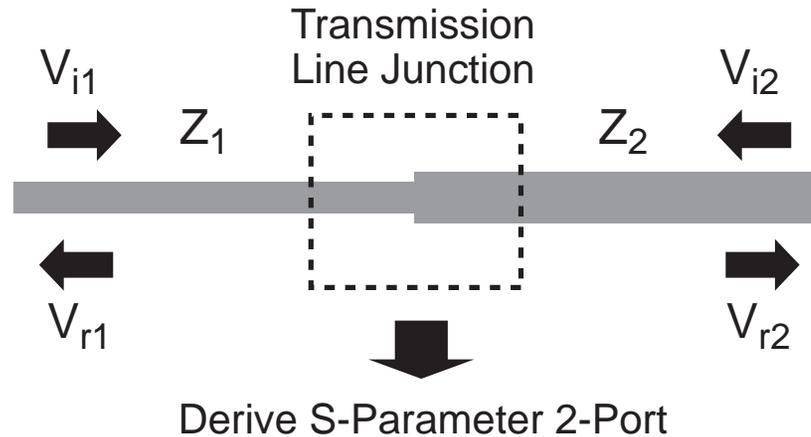


# Note for CppSim Simulations



- **CppSim does block-by-block computation**
  - Feedback introduces artificial delays in simulation
- **Prevent artificial delays by**
  - Ordering blocks according to input-to-output signal flow
  - Creating an additional signal in CppSim modules to pass previous sample values
  - Note: both are already done for you in Homework #1

# S-Parameter Calculations – Example 1



## ■ Set $V_{i2} = 0$

$$V_{r1} = \Gamma_1 V_{i1} = \frac{Z_2 - Z_1}{Z_2 + Z_1} V_{i1}$$

$$V_{r2} = V_{i1} + V_{r1} = (1 + \Gamma_1) V_{i1}$$

## ■ Set $V_{i1} = 0$

$$V_{r2} = \Gamma_2 V_{i2} = \frac{Z_1 - Z_2}{Z_1 + Z_2} V_{i2}$$

$$V_{r1} = V_{i2} + V_{r2} = (1 + \Gamma_2) V_{i2}$$

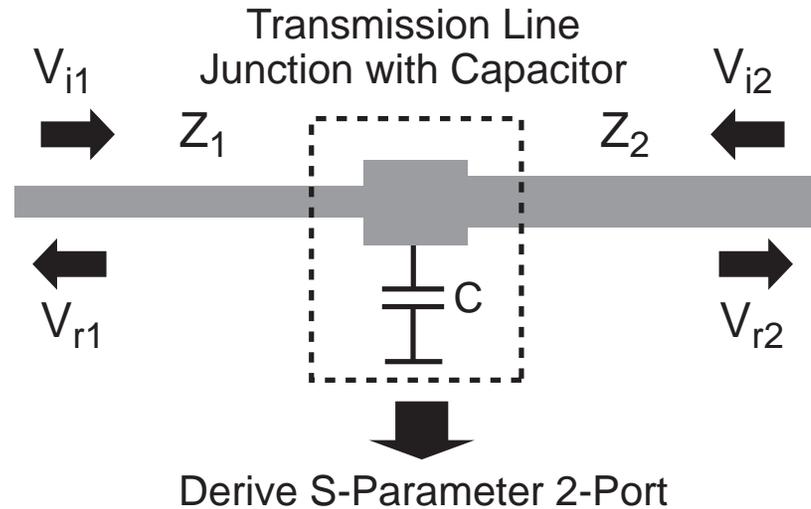
$$\Rightarrow S_{11} = \Gamma_1$$

$$\Rightarrow S_{21} = \sqrt{\frac{Z_1}{Z_2}} (1 + \Gamma_1)$$

$$\Rightarrow S_{22} = \Gamma_2$$

$$\Rightarrow S_{12} = \sqrt{\frac{Z_2}{Z_1}} (1 + \Gamma_2)$$

## S-Parameter Calculations – Example 2



- Same as before:

$$\Rightarrow S_{11} = \Gamma_1$$

$$\Rightarrow S_{22} = \Gamma_2$$

$$\Rightarrow S_{21} = \sqrt{\frac{Z_1}{Z_2}}(1 + \Gamma_1)$$

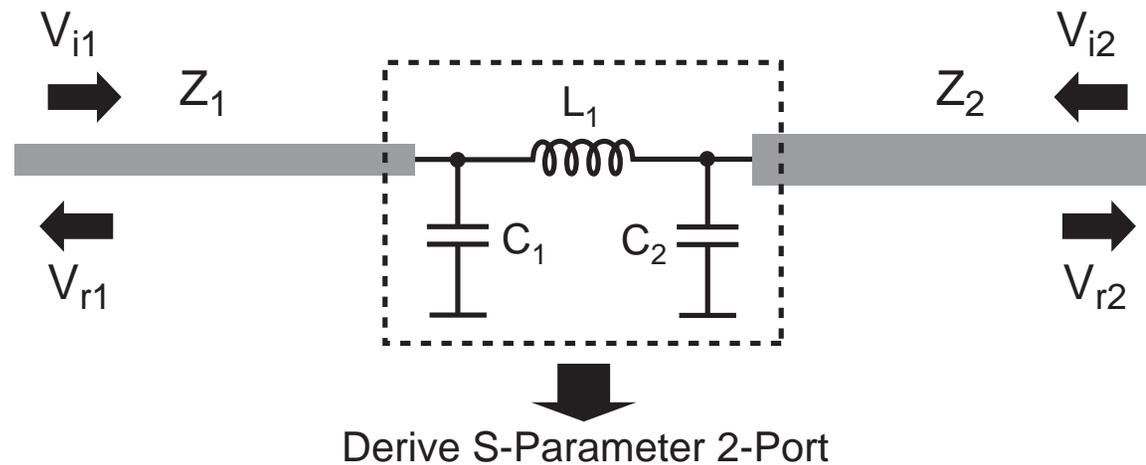
$$\Rightarrow S_{12} = \sqrt{\frac{Z_2}{Z_1}}(1 + \Gamma_2)$$

- But now:

$$\Gamma_1 = \frac{Z_2 \parallel (1/sC) - Z_1}{Z_2 \parallel (1/sC) + Z_1}$$

$$\Gamma_2 = \frac{Z_1 \parallel (1/sC) - Z_2}{Z_1 \parallel (1/sC) + Z_2}$$

## S-Parameter Calculations – Example 3

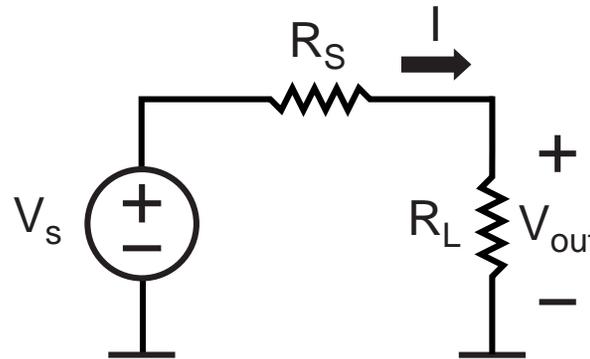


- The S-parameter calculations are now more involved
  - Network now has more than one node
- This is a homework problem

# ***Impedance Transformers***

# Matching for Voltage versus Power Transfer

- Consider the voltage divider network



- For maximum voltage transfer

$$R_L \rightarrow \infty \Rightarrow V_{out} \rightarrow V_s$$

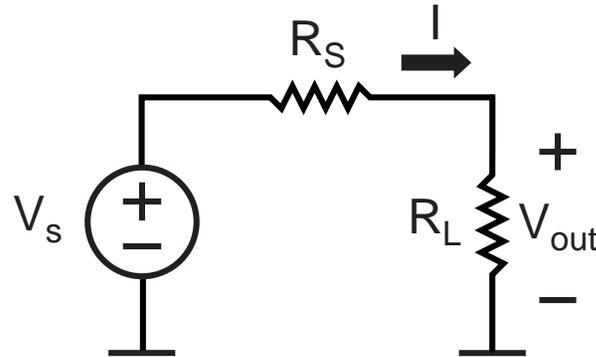
- For maximum power transfer

$$R_L = R_S \Rightarrow P_{out} = \frac{|V_{out}|^2}{R_L} = \frac{|V_s|^2}{4R_S}$$

**Which one do we want?**

## Note: Maximum Power Transfer Derivation

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- **Formulation**

$$P_{out} = I^2 R_L = \left( \frac{V_s}{R_S + R_L} \right)^2 R_L = \frac{R_L}{(R_S + R_L)^2} V_s^2$$

- **Take the derivative and set it to zero**

$$\frac{dP_{out}}{dR_L} = R_L(-2)(R_S + R_L)^{-3} + (R_S + R_L)^{-2} = 0$$

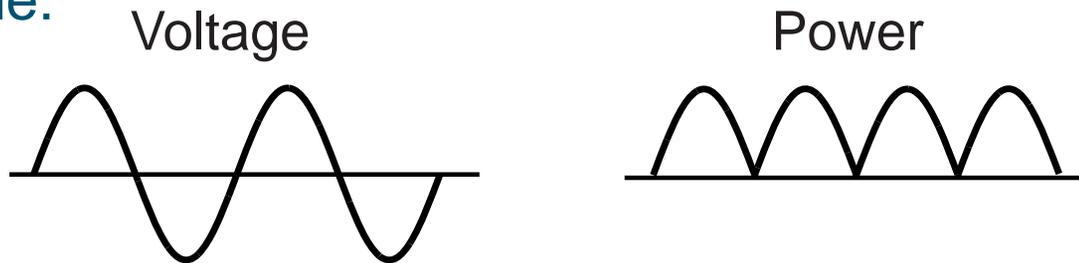
$$\Rightarrow 2R_L = R_S + R_L \Rightarrow R_L = R_S$$

# Voltage Versus Power

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- For most communication circuits, voltage (or current) is the key signal for detection
  - Phase information is important
  - Power is ambiguous with respect to phase information

- Example:

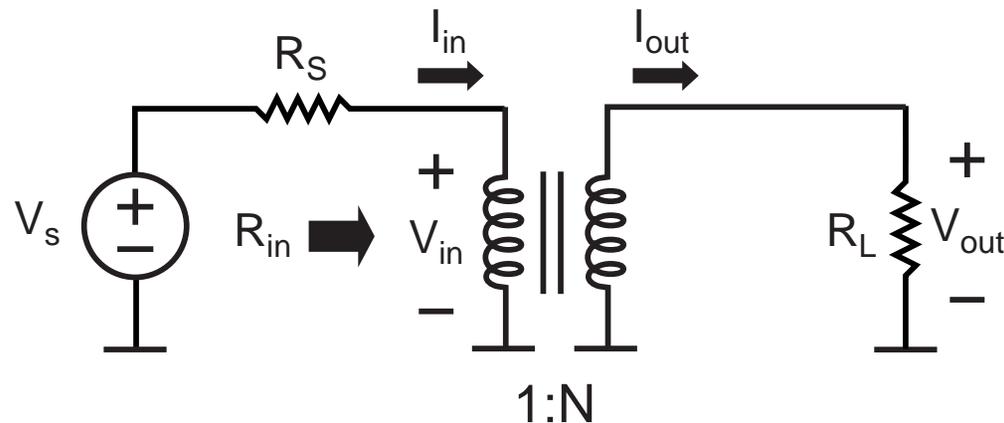


- For high speed circuits with transmission lines, achieving maximum power transfer is important
  - Maximum power transfer coincides with having zero reflections (i.e.,  $\Gamma_L = 0$ )

Can we ever win on both issues?

# Broadband Impedance Transformers

- Consider placing an ideal transformer between source and load



- Transformer basics (passive, zero loss)

1)  $V_{out} = NV_{in}$

2) Power In = Power Out

$$\Rightarrow V_{in}I_{in} = V_{out}I_{out}$$

From (1) and (2):  $V_{in}I_{in} = NV_{in}I_{out} \Rightarrow I_{out} = \frac{I_{in}}{N}$

- Transformer input impedance

$$R_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{out}/N}{NI_{out}} = \frac{1}{N^2}R_L$$

## What Value of $N$ Maximizes Voltage Transfer?

- Derive formula for  $V_{out}$  versus  $V_{in}$  for given  $N$  value

$$\begin{aligned}V_{out} = NV_{in} &= N \frac{R_{in}}{R_s + R_{in}} V_s = N \frac{R_L/N^2}{R_s + R_L/N^2} V_s \\ &= N \frac{R_L}{R_L + N^2 R_s} V_s\end{aligned}$$

- Take the derivative and set it to zero

$$\frac{dV_{out}}{dN} = NR_L(-1)(R_L + N^2 R_S)^{-2} 2NR_S + R_L(R_L + N^2 R_S)^{-1} = 0$$

$$\Rightarrow -2N^2 R_S (R_L + N^2 R_S)^{-2} + (R_L + N^2 R_S)^{-1} = 0$$

$$\Rightarrow -2N^2 R_S = R_L + N^2 R_S \quad \Rightarrow \quad \boxed{N^2 = \frac{R_L}{R_S}}$$

## What is the Input Impedance for Max Voltage Transfer?

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- We know from basic transformer theory that input impedance into transformer is

$$R_{in} = \frac{1}{N^2} R_L$$

- We just learned that, to maximize voltage transfer, we must set the transformer turns ratio to

$$N^2 = \frac{R_L}{R_s}$$

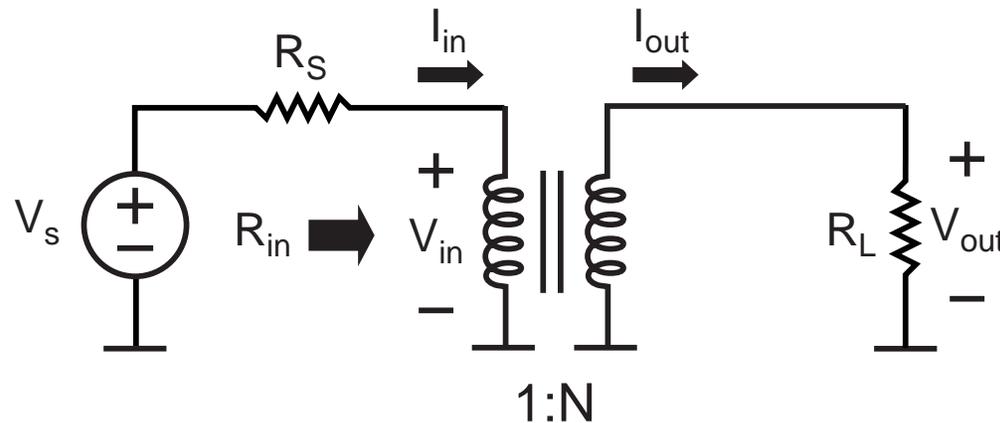
- Put them together

$$R_{in} = \frac{1}{N^2} R_L = \frac{1}{R_L/R_s} R_L = R_s \quad !!$$

**So, N should be set for max power transfer into transformer to achieve the maximum voltage transfer at the load!**

# Benefit of Impedance Matching with Transformers

- Transformers allow maximum voltage and power transfer relationship to coincide



- Turns ratio for max power/voltage transfer

$$N^2 = \frac{R_L}{R_s}$$

- Resulting voltage gain (can exceed one!)

$$V_{out} = NV_{in} = N \left( \frac{1}{2} V_s \right) = \sqrt{\frac{R_L}{R_s}} \left( \frac{1}{2} V_s \right)$$

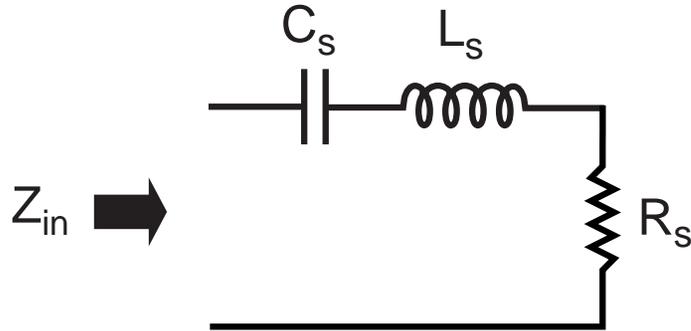
## ***The Catch***

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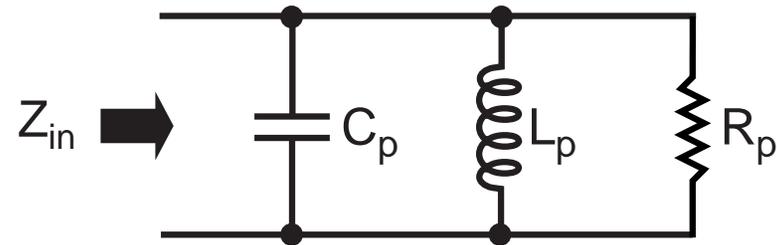
- **It's hard to realize a transformer with good performance over a wide frequency range**
  - **Magnetic materials have limited frequency response**
  - **Inductors have self-resonant frequencies, losses, and mediocre coupling to other inductors without magnetic material**
  
- **For wireless applications, we only need transformers that operate over a small frequency range**
  - **Can we take advantage of this?**

# Consider Resonant Circuits (Chap. 4 of Lee's Book)

Series Resonant Circuit



Parallel Resonant Circuit



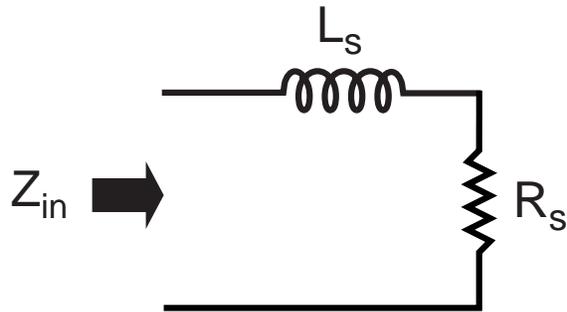
$$\begin{aligned} Z_{in} &= \frac{1}{j\omega C_s} + j\omega L_s + R_s \\ &= R_s \text{ for } \omega = \frac{1}{\sqrt{L_s C_s}} = \omega_o \\ Q &= \frac{\omega_o L_s}{R_s} = \frac{1}{\omega_o C_s R_s} \end{aligned}$$

$$\begin{aligned} Z_{in} &= \frac{1}{j\omega C_p} || j\omega L_p || R_p \\ &= R_p \text{ for } \omega = \frac{1}{\sqrt{L_p C_p}} = \omega_o \\ Q &= \frac{R_p}{\omega_o L_p} = \omega_o C_p R_p \end{aligned}$$

- **Key insight: resonance allows  $Z_{in}$  to be purely real despite the presence of reactive elements**

# Comparison of Series and Parallel RL Circuits

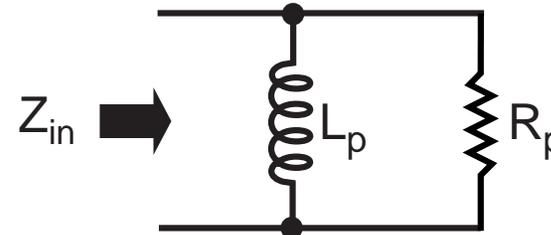
Series RL Circuit



$$Q = \frac{\omega_0 L_s}{R_s}$$

$$Z_{in} = j\omega_0 L_s + R_s$$

Parallel RL Circuit



$$Q = \frac{R_p}{\omega_0 L_p}$$

$$Z_{in} = j\omega_0 L_p || R_p$$

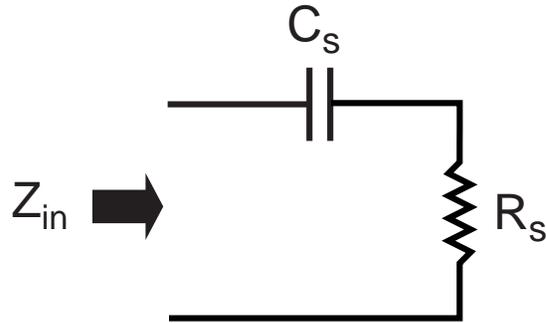
- **Equate real and imaginary parts of the left and right expressions (so that  $Z_{in}$  is the same for both)**
  - **Also equate Q values**

$$R_p = R_s(Q^2 + 1) \approx R_s Q^2 \quad (\text{for } Q \gg 1)$$

$$L_p = L_s \left( \frac{Q^2 + 1}{Q^2} \right) \approx L_s \quad (\text{for } Q \gg 1)$$

# Comparison of Series and Parallel RC Circuits

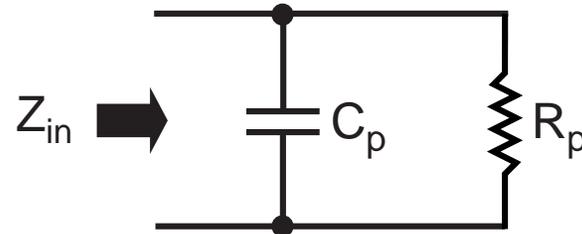
Series RC Circuit



$$Q = \frac{1}{\omega_0 C_s R_s}$$

$$Z_{in} = R_s + \frac{1}{j\omega_0 C_s}$$

Parallel RC Circuit



$$Q = \omega_0 C_p R_p$$

$$Z_{in} = R_p \parallel \frac{1}{j\omega_0 C_p}$$

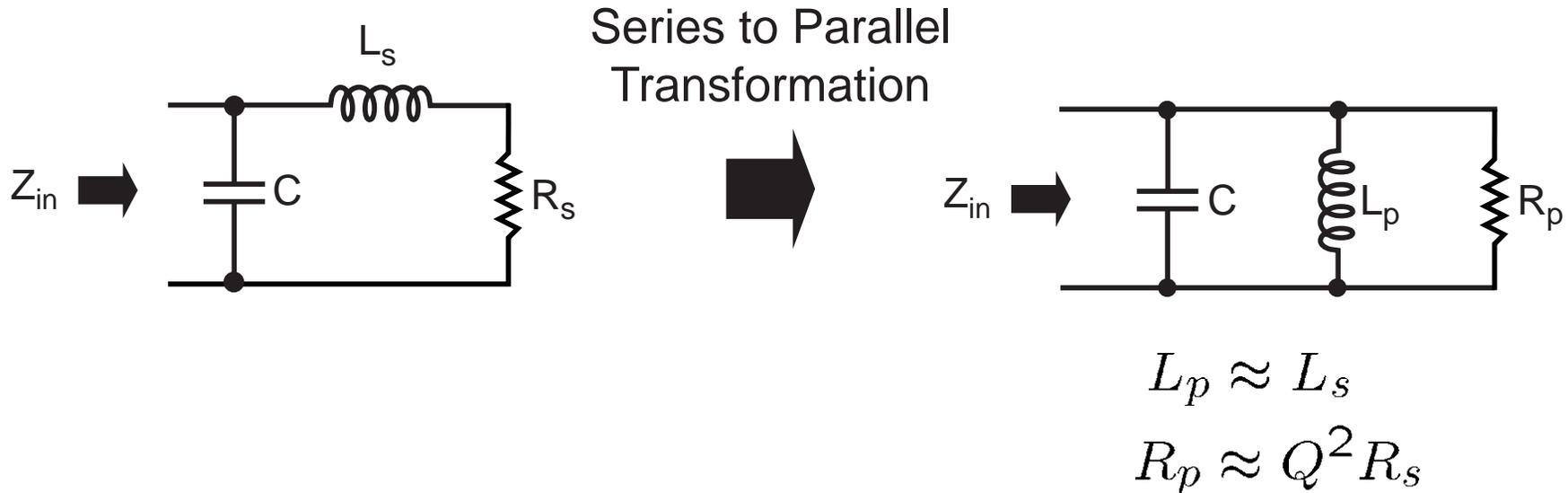
- **Equate real and imaginary parts of the left and right expressions (so that  $Z_{in}$  is the same for both)**
  - **Also equate Q values**

$$R_p = R_s(Q^2 + 1) \approx R_s Q^2 \quad (\text{for } Q \gg 1)$$

$$C_p = C_s \left( \frac{Q^2}{Q^2 + 1} \right) \approx C_s \quad (\text{for } Q \gg 1)$$

# A Narrowband Transformer: The L Match

- Assume  $Q \gg 1$



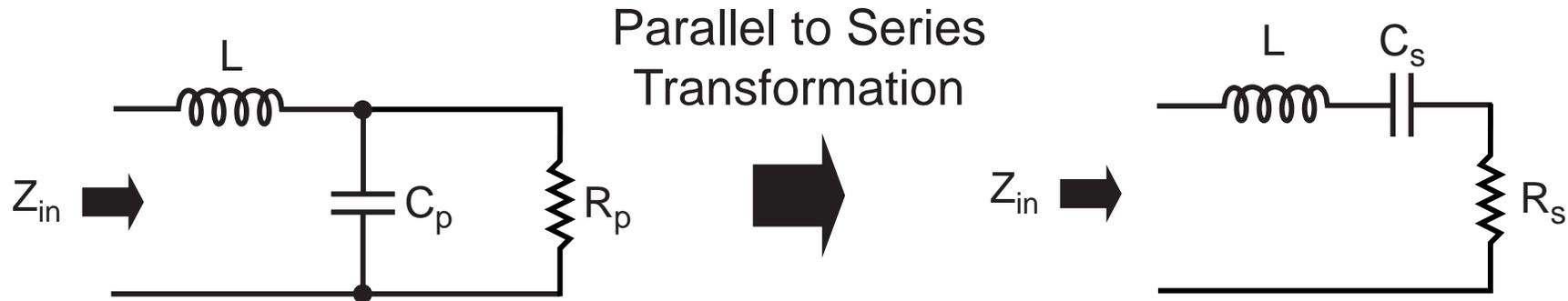
- So, at resonance

$$Z_{in} = R_p \approx Q^2 R_s \quad (\text{purely real})$$

- Transformer steps up impedance!

# Alternate Implementation of L Match

- Assume  $Q \gg 1$



$$C_s \approx C_p$$
$$R_s \approx \frac{R_p}{Q^2}$$

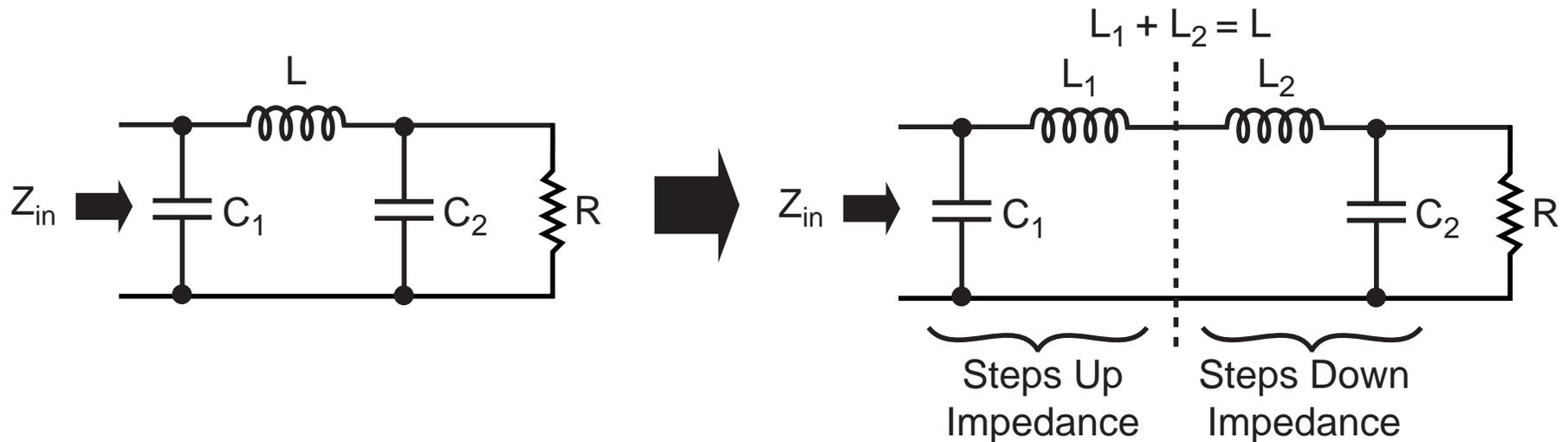
- So, at resonance

$$Z_{in} = R_s \approx \frac{R_p}{Q^2} \quad (\text{purely real})$$

- Transformer steps down impedance!

# The $\pi$ Match

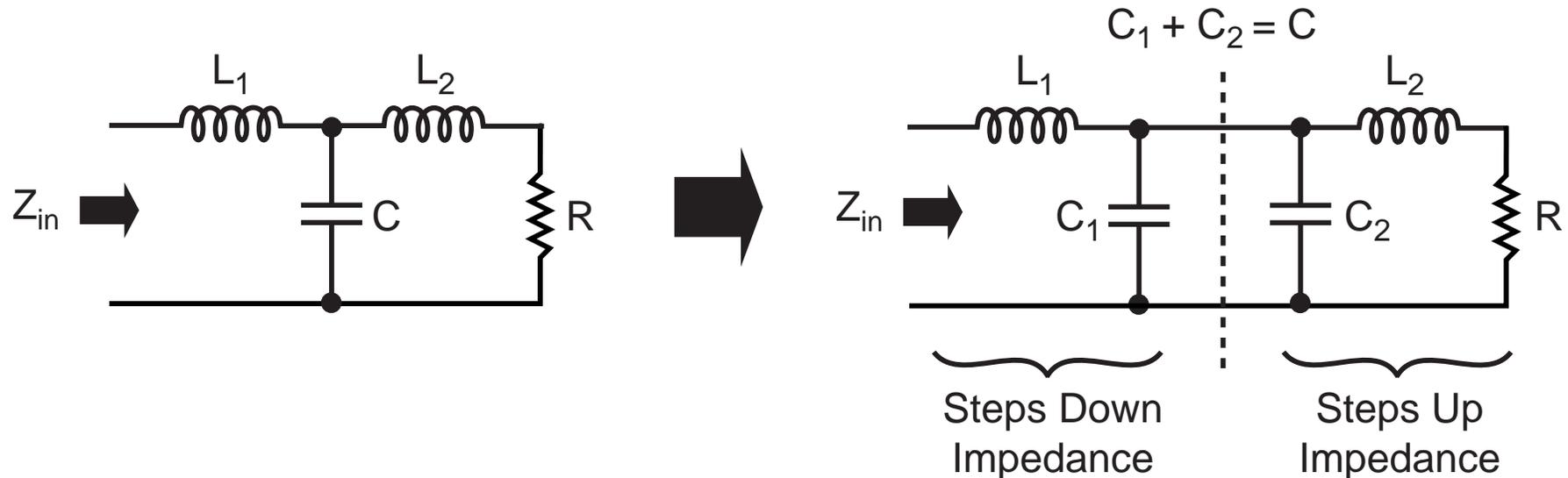
- Combines two L sections



- Provides an extra degree of freedom for choosing component values for a desired transformation ratio

# The T Match

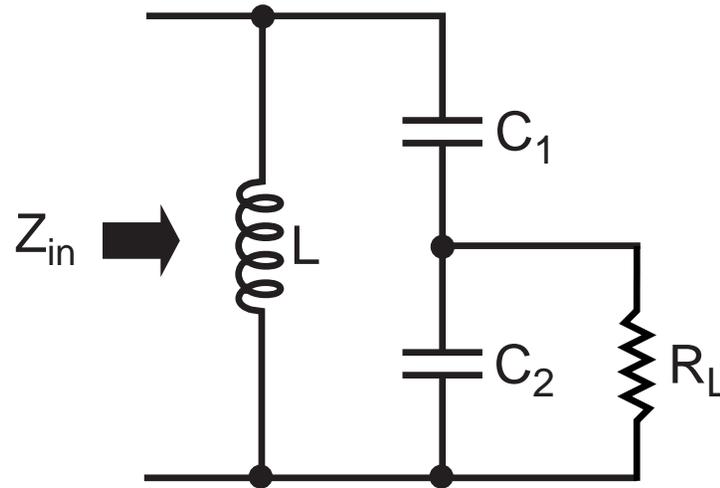
- Also combines two L sections



- Again, benefit is in providing an extra degree of freedom in choosing component values

# Tapped Capacitor as a Transformer

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- To first order:

$$\frac{R_{in}}{R_L} \approx \left( \frac{C_1 + C_2}{C_1} \right)^2$$

- Useful in VCO design
- See Chapter 4 of Tom Lee's book for analysis