

High Speed Communication Circuits and Systems
Lecture 20
Performance Measures of Wireless Communication

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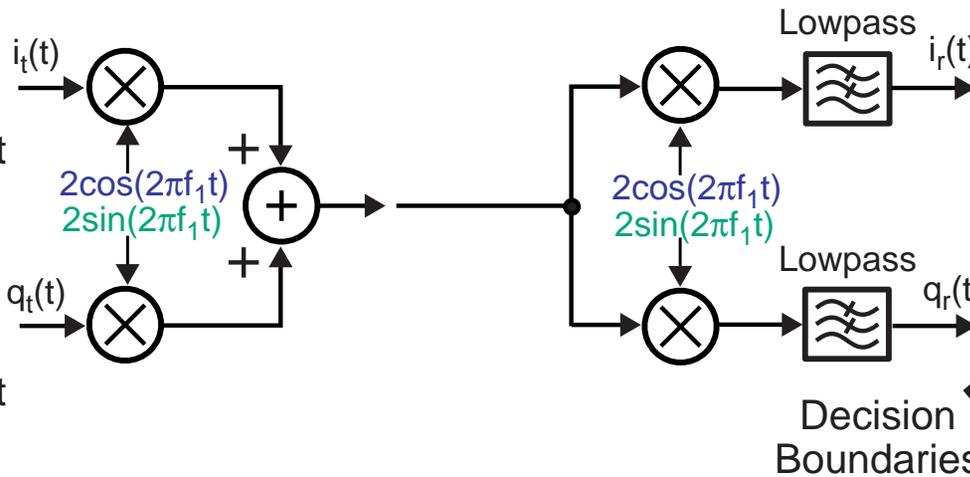
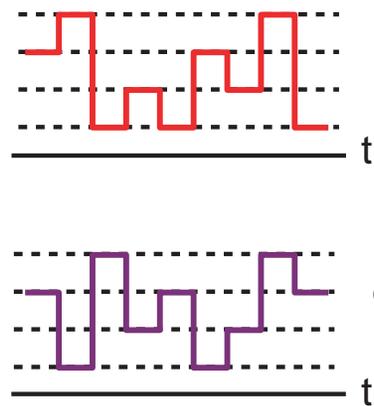
April 21, 2004

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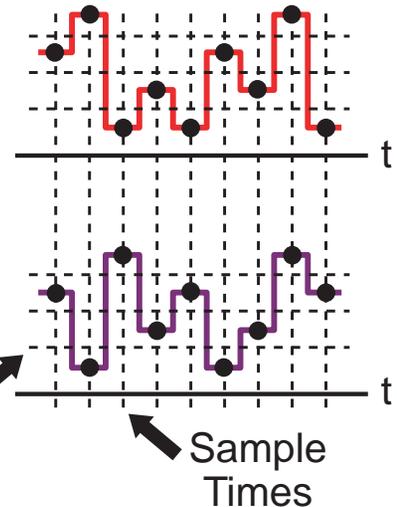
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Recall Digital Modulation for Wireless Link

Baseband Input

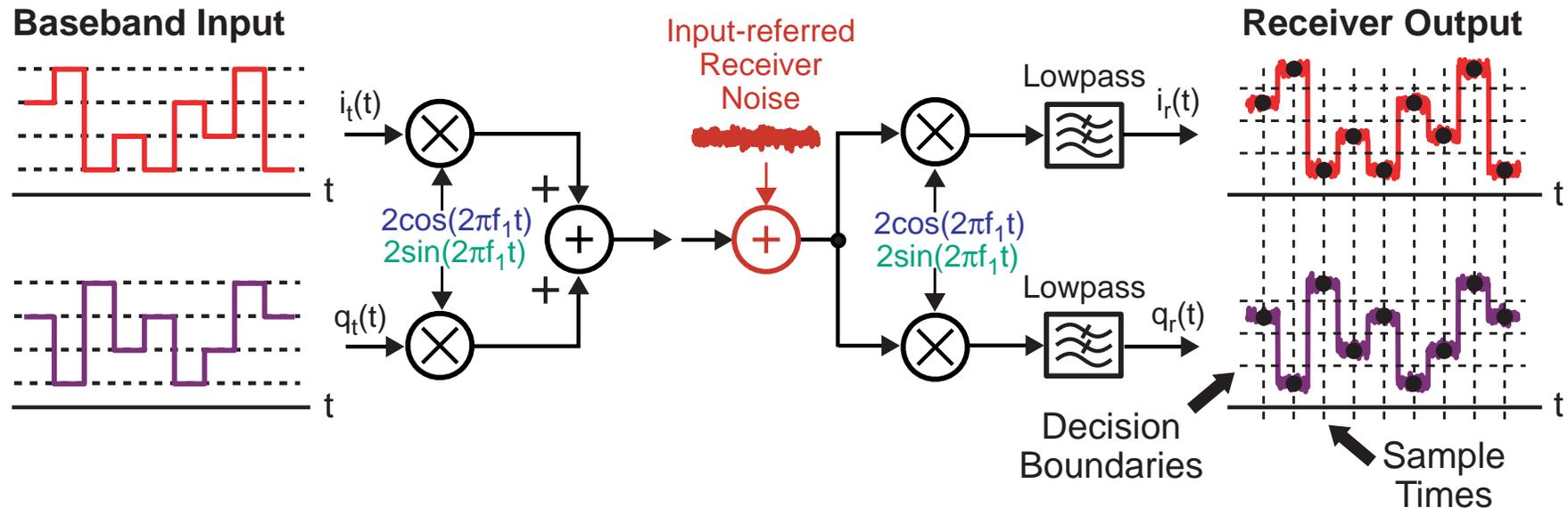


Receiver Output



- Send discrete-leveled values on I and Q channels
- Performance issues
 - Spectral efficiency (transmitter)
 - Bit error rate performance (receiver)
- Nonidealities
 - Intersymbol interference
 - Noise
 - Interferers

Impact of Receiver Noise



■ Performance impact

- SNR is reduced, leading to possible bit errors

■ Methods of increasing SNR

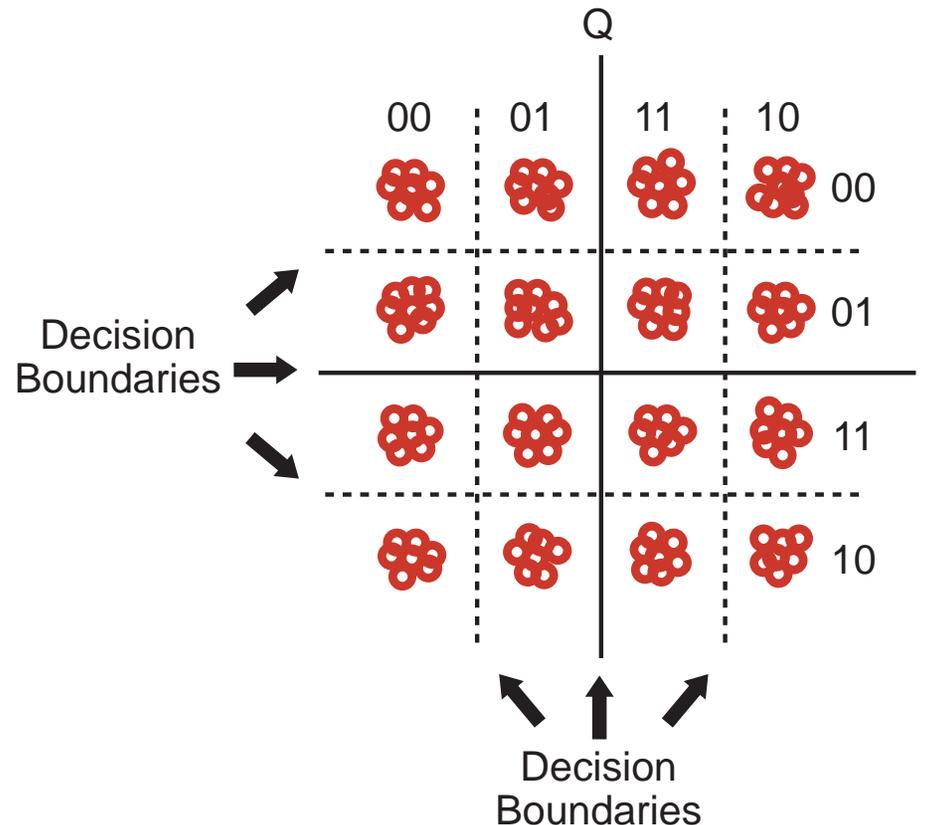
- Decrease bandwidth of receiver lowpass

- SNR is traded off for intersymbol interference

- Increase input power into receiver

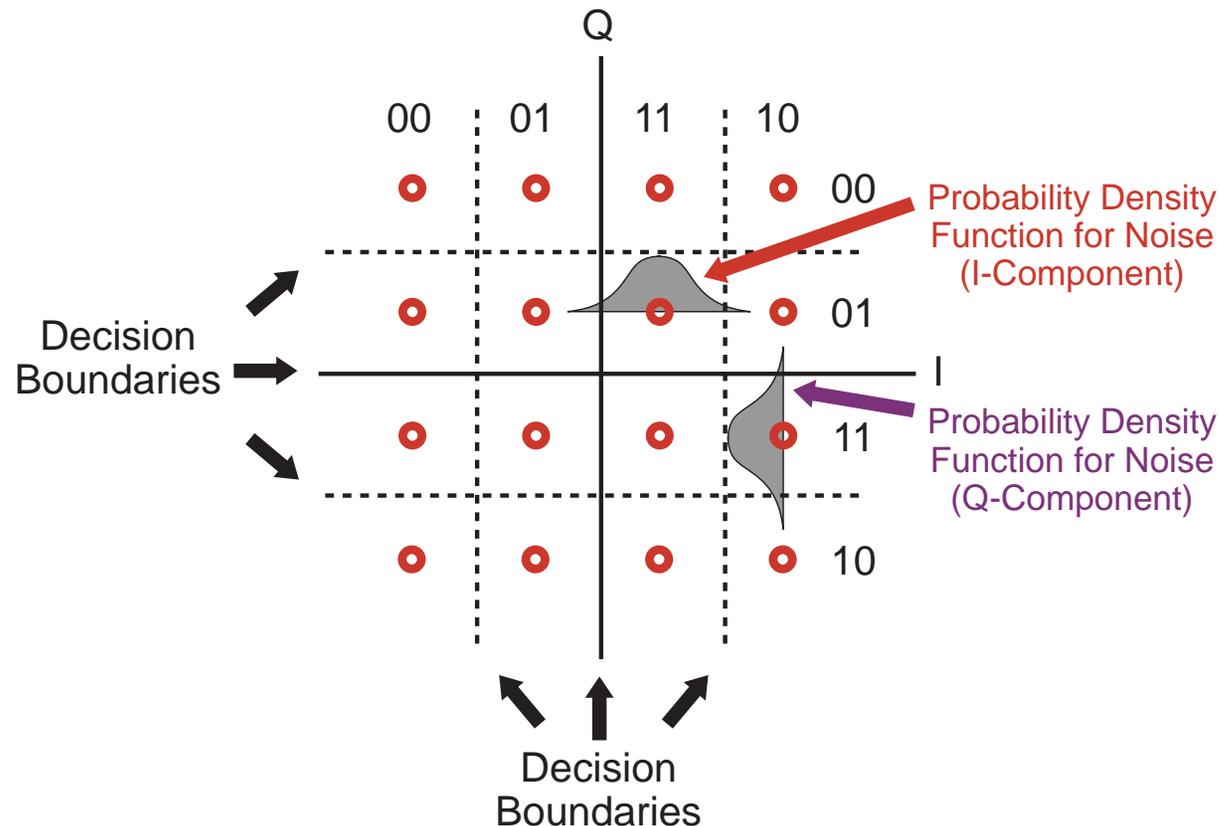
- Increase transmit power and/or shorten its distance from receiver

View SNR Issue with Constellation Diagram



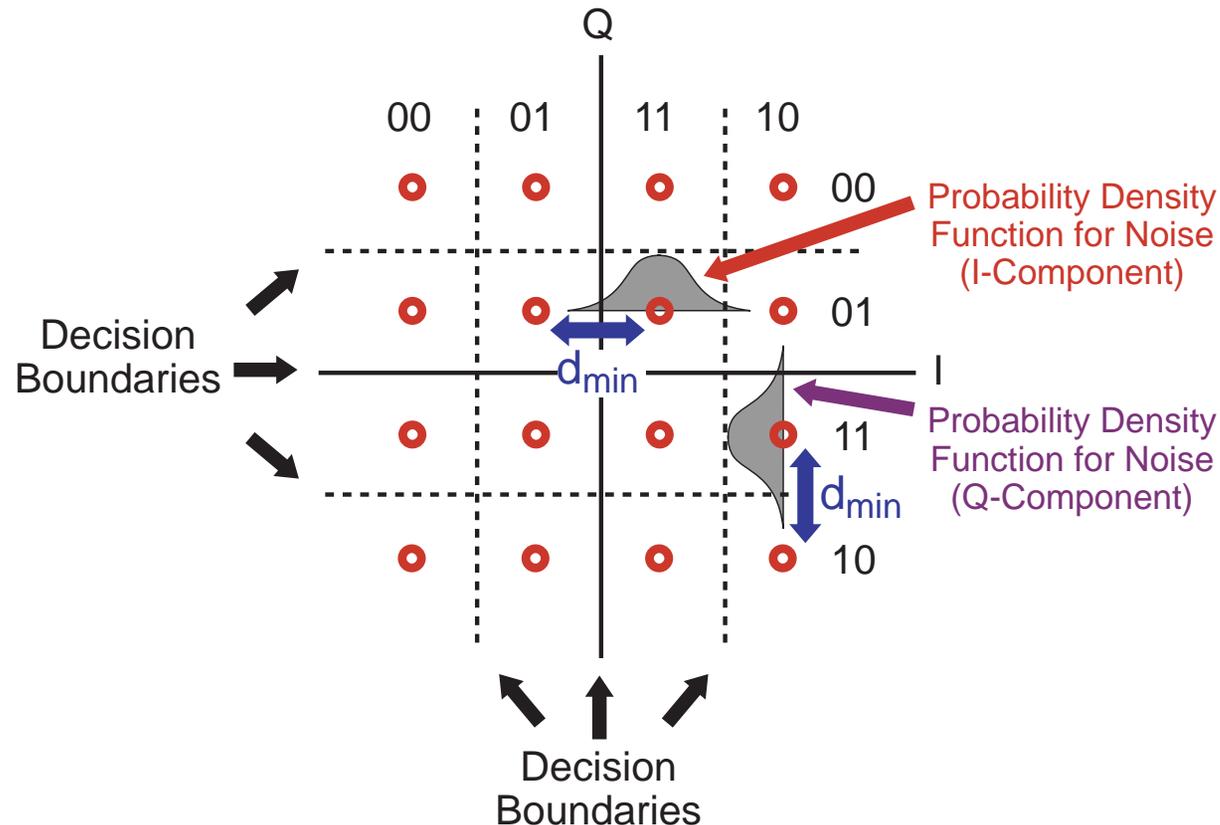
- **Noise causes sampled I/Q values to vary about their nominal values**
- **Bit errors are created when sampled I/Q values cross decision boundaries**

Mathematical Analysis of SNR versus Bit Error Rate (BER)



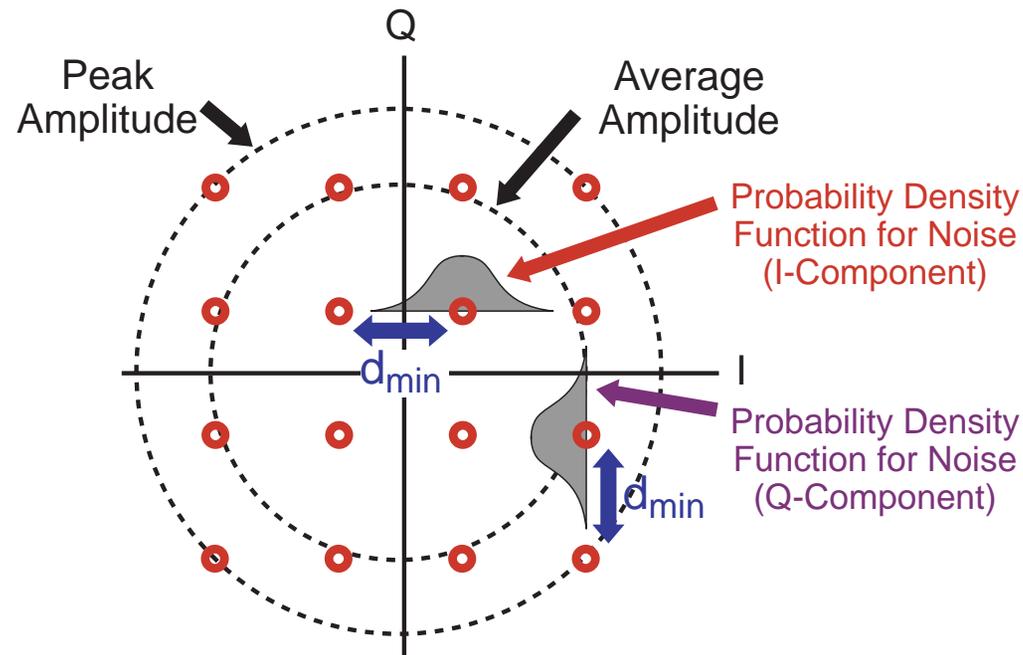
- **Model noise impacting I and Q channels according to a probability density function (PDF)**
 - Gaussian shape is often assumed
- **Receiver bit error rate can be computed by calculating probability of tail regions of PDF curves**

Key Parameters for SNR/BER Analysis



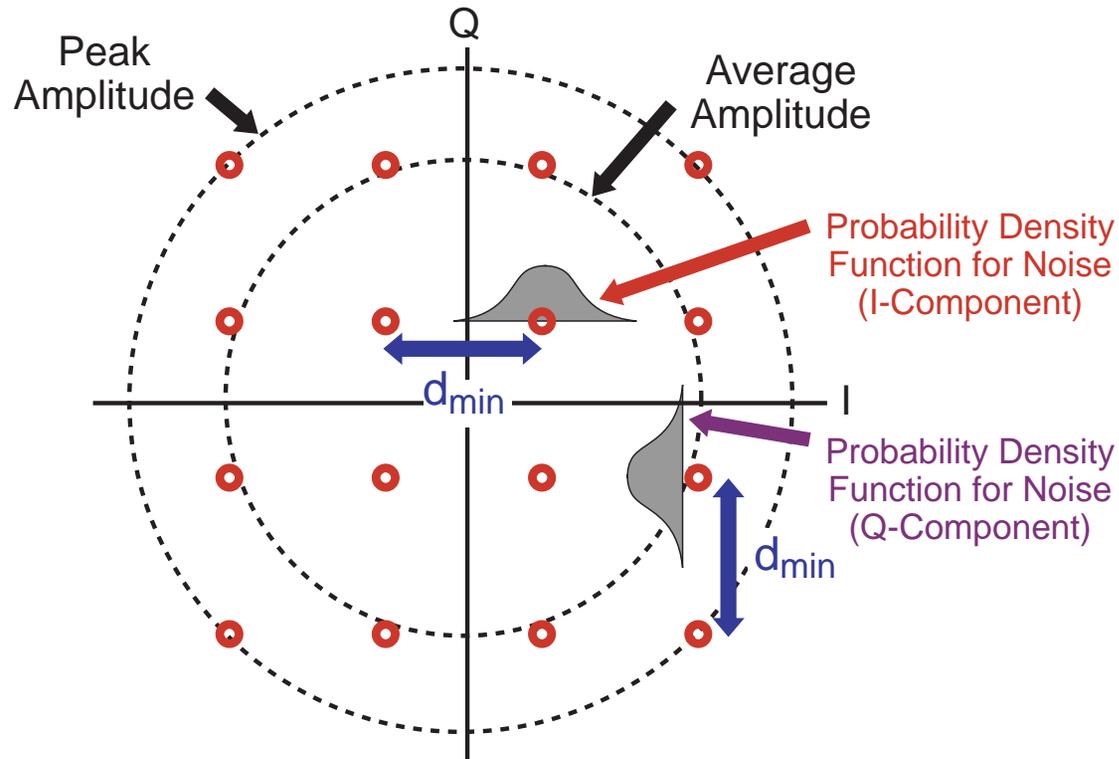
- **Bit error rate (BER) is a function of**
 - Variance of noise
 - Distance between constellation points (d_{\min})
- **Larger d_{\min} with a fixed noise variance leads to higher SNR and a lower bit error rate**

Relationship Between Amplitude and Constellation



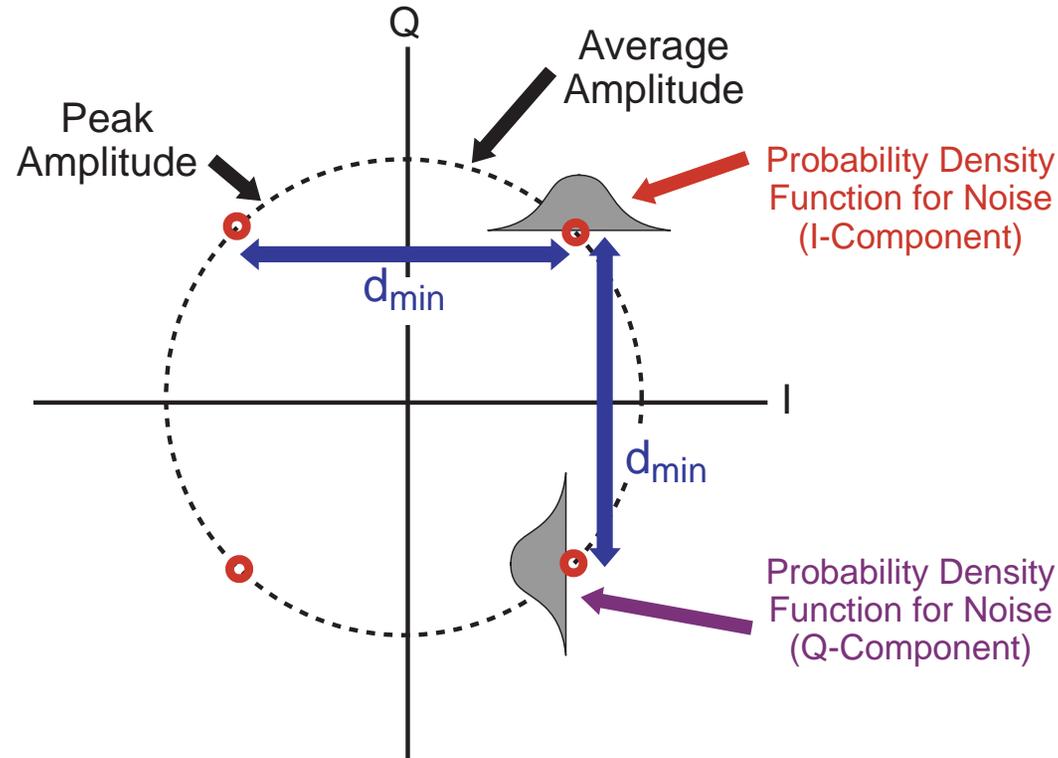
- Distance of I/Q constellation point from origin corresponds to instantaneous amplitude of input signal at that sample time
- Amplitude is measured at receiver and a function of
 - Transmit power
 - Distance between transmitter and receiver (and channel)

Impact of Increased Signal Power At Receiver



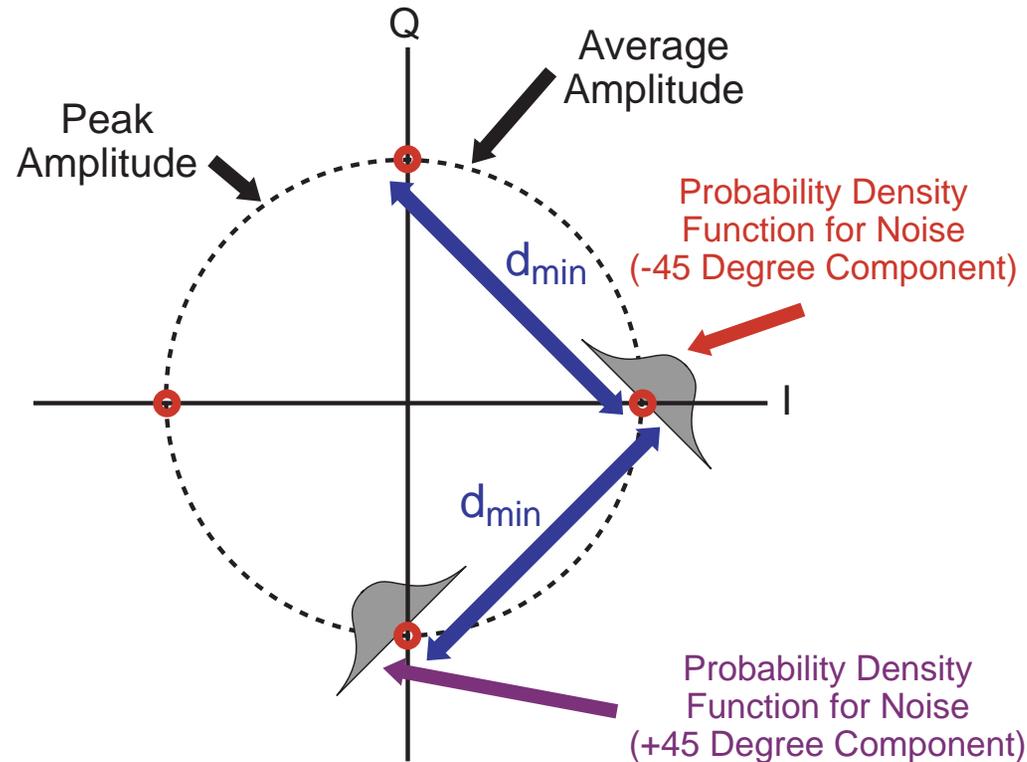
- Separation between constellation points, d_{min} , increases as received power increases
- Noise variance remains roughly constant as input signal power is increased
 - Noise variance primarily determined by receiver circuits
- Bit error rate improves with increased signal power!

Impact of Modulation Scheme on SNR/BER



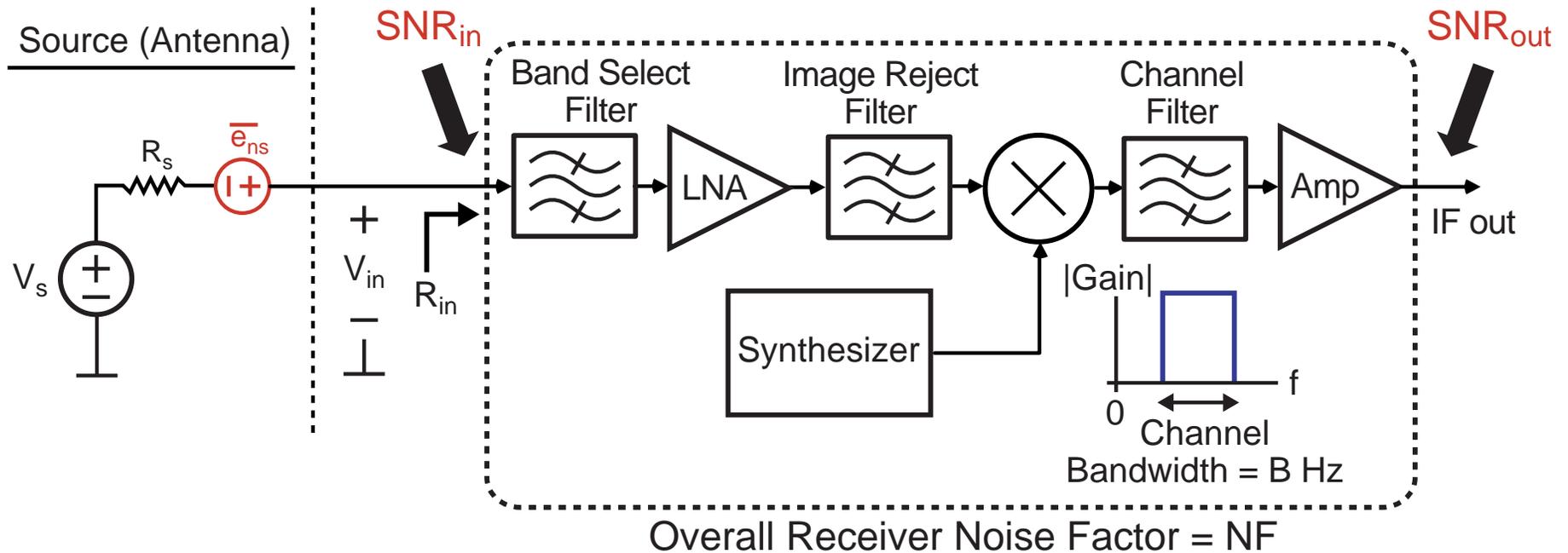
- Lowering the number of constellation points increases d_{min} for a fixed input signal amplitude
 - SNR is increased (given a fixed noise variance)
 - Bit error is reduced
- Actual situation is more complicated when coding is used

Alternate View of Previous Constellation



- Common modulation method is to transmit independent binary signals on I and Q channels
- The above constellation has the same d_{\min} as the one on the previous slide
 - Obtains the same SNR/BER performance given that the noise on I/Q channels is symmetric

Impact of Noise Factor on Input-Referred SNR



$$NF = \frac{SNR_{in}}{SNR_{out}}$$

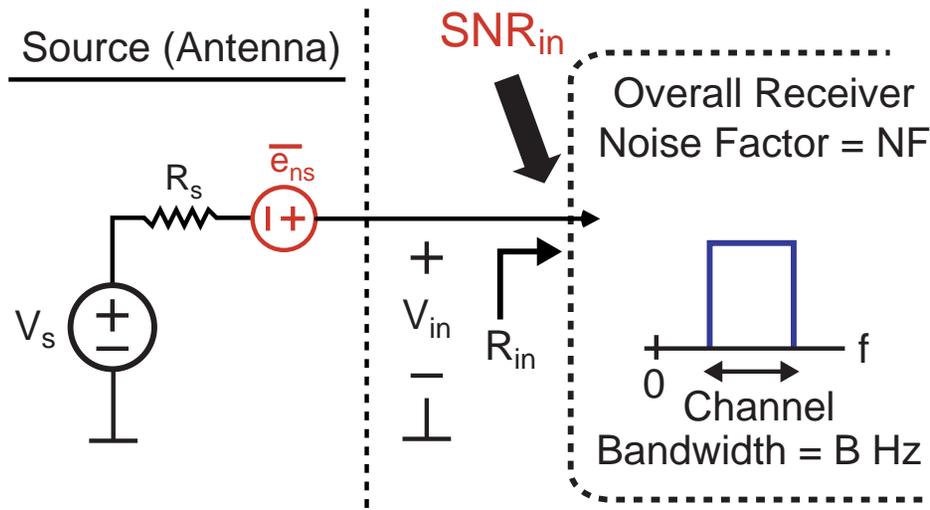
- To achieve acceptable bit error rates (BER)

$$SNR_{out} \geq SNR_{out,min}$$

- Refer SNR requirement to input

$$SNR_{in} = NF \cdot SNR_{out} \geq NF \cdot SNR_{out,min}$$

Minimum Input Power to Achieve Acceptable SNR



$$SNR_{in,min} = NF \cdot SNR_{out,min} \quad (1)$$

- Calculation of input SNR in terms of input power

$$SNR_{in} = \frac{v_{in}^2 / R_{in}}{\alpha^2 e_{nRs}^2 / R_{in}} = \frac{P_{in}}{\alpha^2 e_{nRs}^2 / R_{in}}, \quad \text{where } \alpha = \frac{R_{in}}{R_s + R_{in}} \quad (2)$$

- Combine (1) and (2)

$$P_{in,min} = \alpha^2 e_{nRs}^2 / R_{in} \cdot NF \cdot SNR_{out,min}$$

Simplified Expression for Minimum Input Power

$$P_{in,min} = \alpha^2 \overline{e_{nR_s}^2} / R_{in} \cdot NF \cdot SNR_{out,min}$$

- Assume that the receiver input impedance is matched to the source (i.e., antenna, etc.)

$$\Rightarrow \alpha = \frac{R_{in}}{R_s + R_{in}} \Big|_{R_{in}=R_s} = \frac{1}{2}$$

$$\Rightarrow \alpha^2 \overline{e_{nR_s}^2} / R_{in} \Big|_{R_{in}=R_s} = \left(\frac{1}{2}\right)^2 \frac{4kTR_s \Delta f}{R_{in}} \Big|_{R_{in}=R_s} = kT \Delta f$$

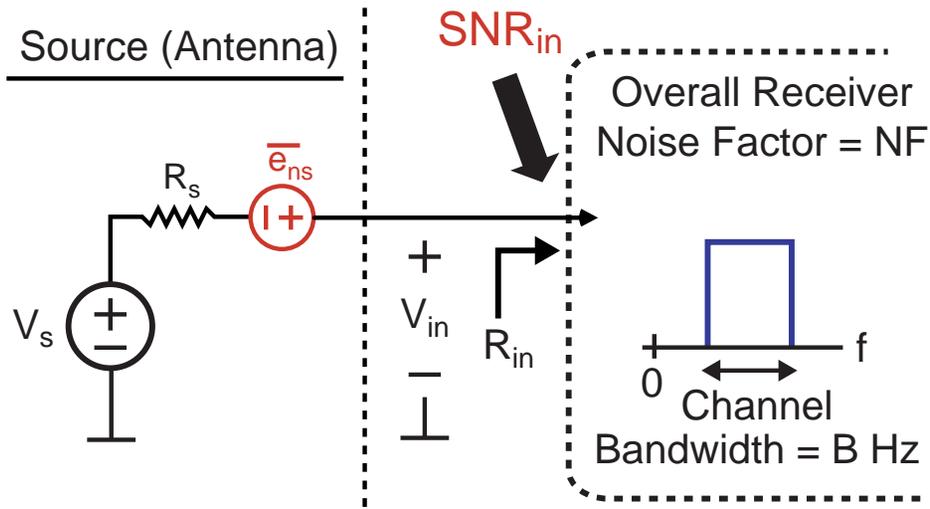
- Resulting expression

$$P_{in,min} = kT \Delta f \cdot NF \cdot SNR_{out,min}$$

- At room temperature:

$$kT = -174 \text{ dBm/Hz}$$

Receiver Sensitivity



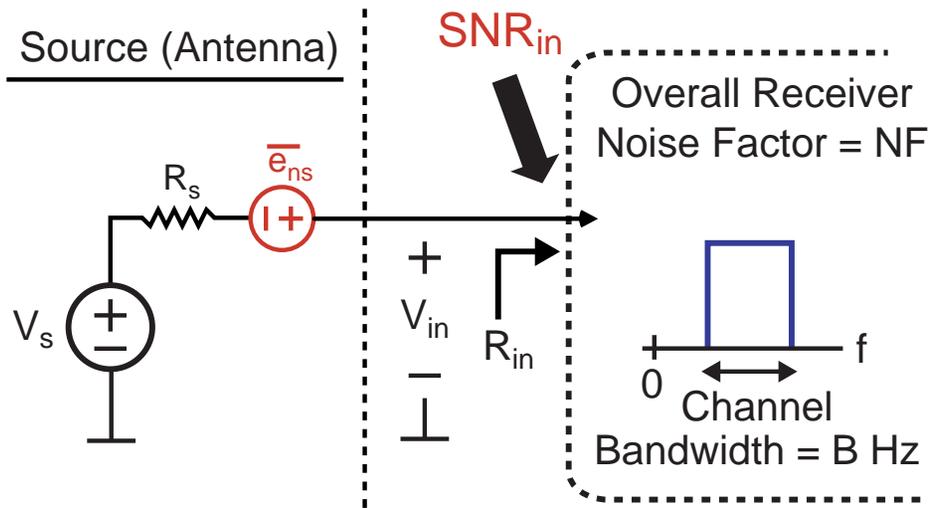
$$P_{in,min} = kT \Delta f \cdot NF \cdot SNR_{out,min}$$

- **Sensitivity of receiver is defined as minimum input power that achieves acceptable SNR (in units of dBm)**

$$dBm(P_{in,min}) = 10 \log(kT \Delta f \cdot NF \cdot SNR_{out,min})$$

$$= -174 + 10 \log(B) + dB(NF) + dB(SNR_{out,min})$$

Example Calculation for Receiver Sensitivity

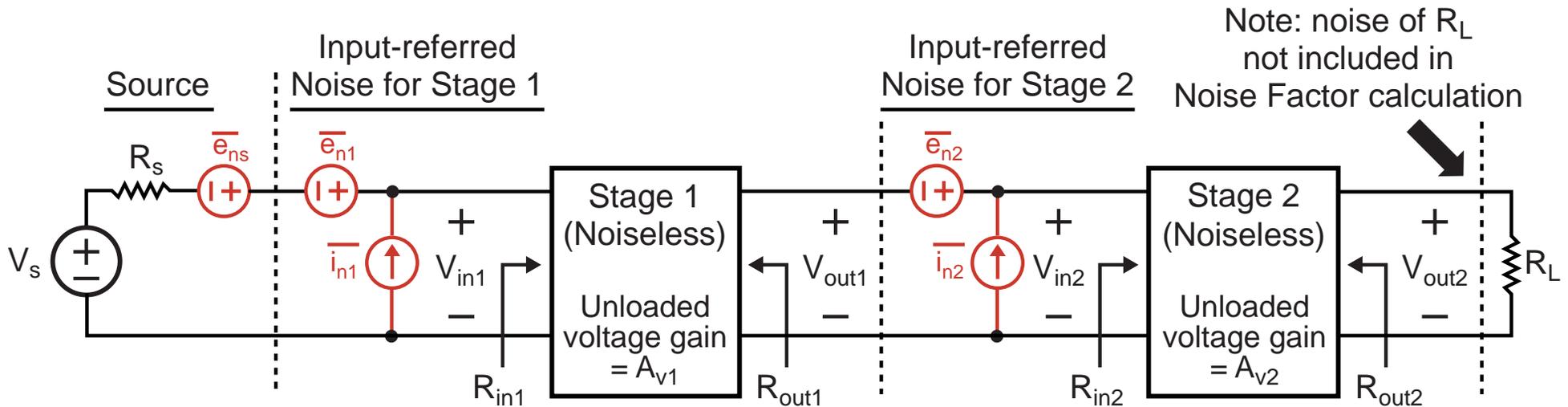


$$dBm(P_{in,min}) = -174 + 10 \log(B) + dB(NF) + dB(SNR_{out,min})$$

- Suppose that a receiver has a noise figure of 8 dB, channel bandwidth is 1 MHz, and the minimum SNR at the receiver output is 12 dB to achieve a BER of 1e-3
 - Receiver sensitivity (for BER of 1e-3) is

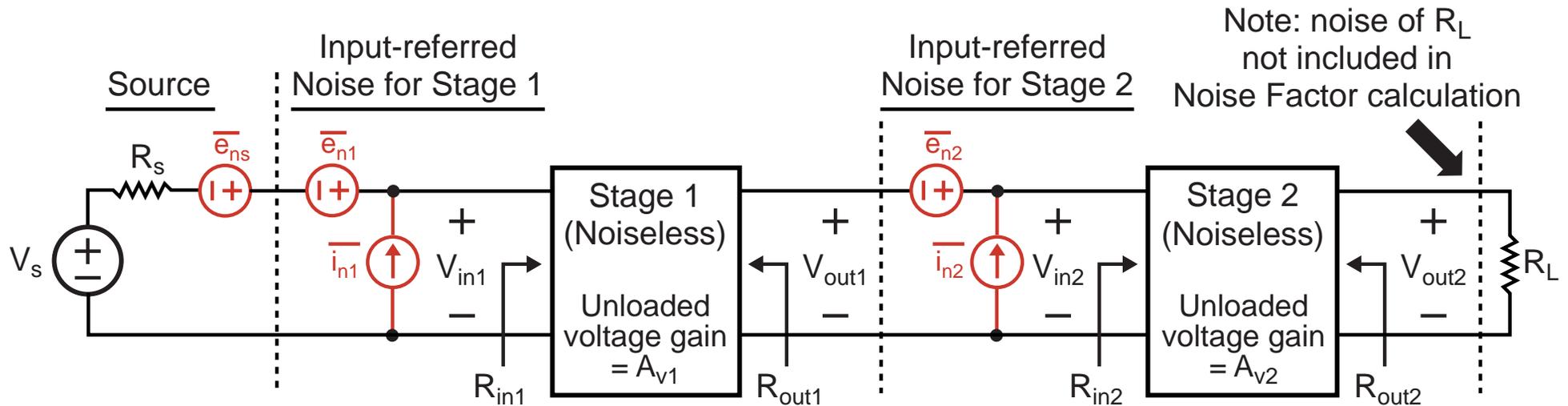
$$dBm(P_{in,min}) = -174 + 60 + 8 + 12 = -94 \text{ dBm}$$

Calculation of Noise Figure of Cascaded Stages

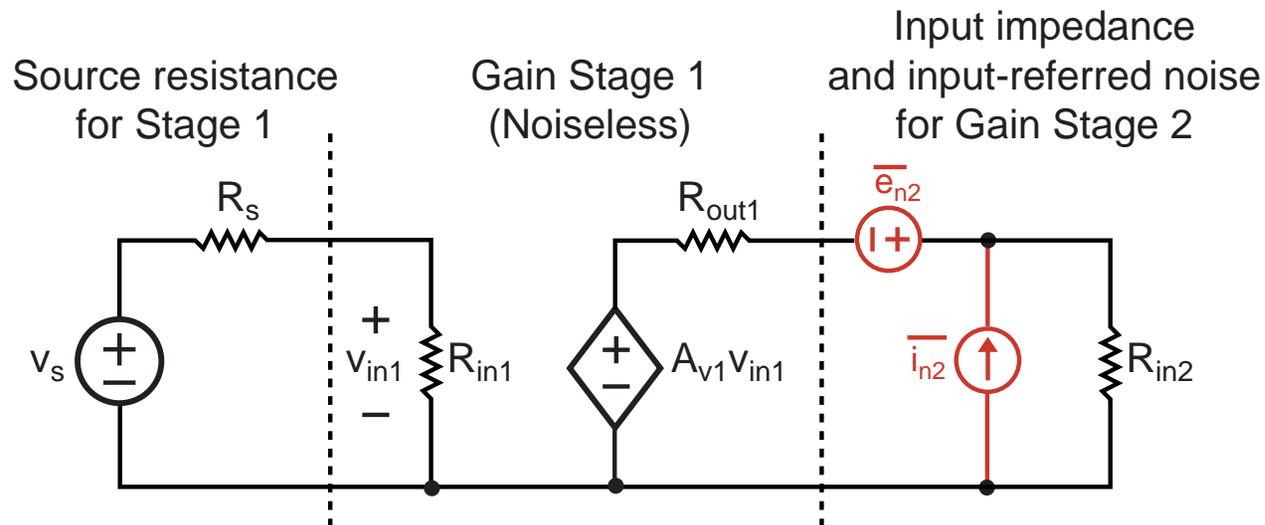


- **Want to calculate overall noise figure of above system**
- **Assumptions**
 - **Input refer the noise sources of each stage**
 - **Model amplification (or attenuation) of each stage as a noiseless voltage controlled voltage source with an unloaded gain equal to A_v**
 - **Ignore noise of final load resistor (or could input refer to previous stage)**

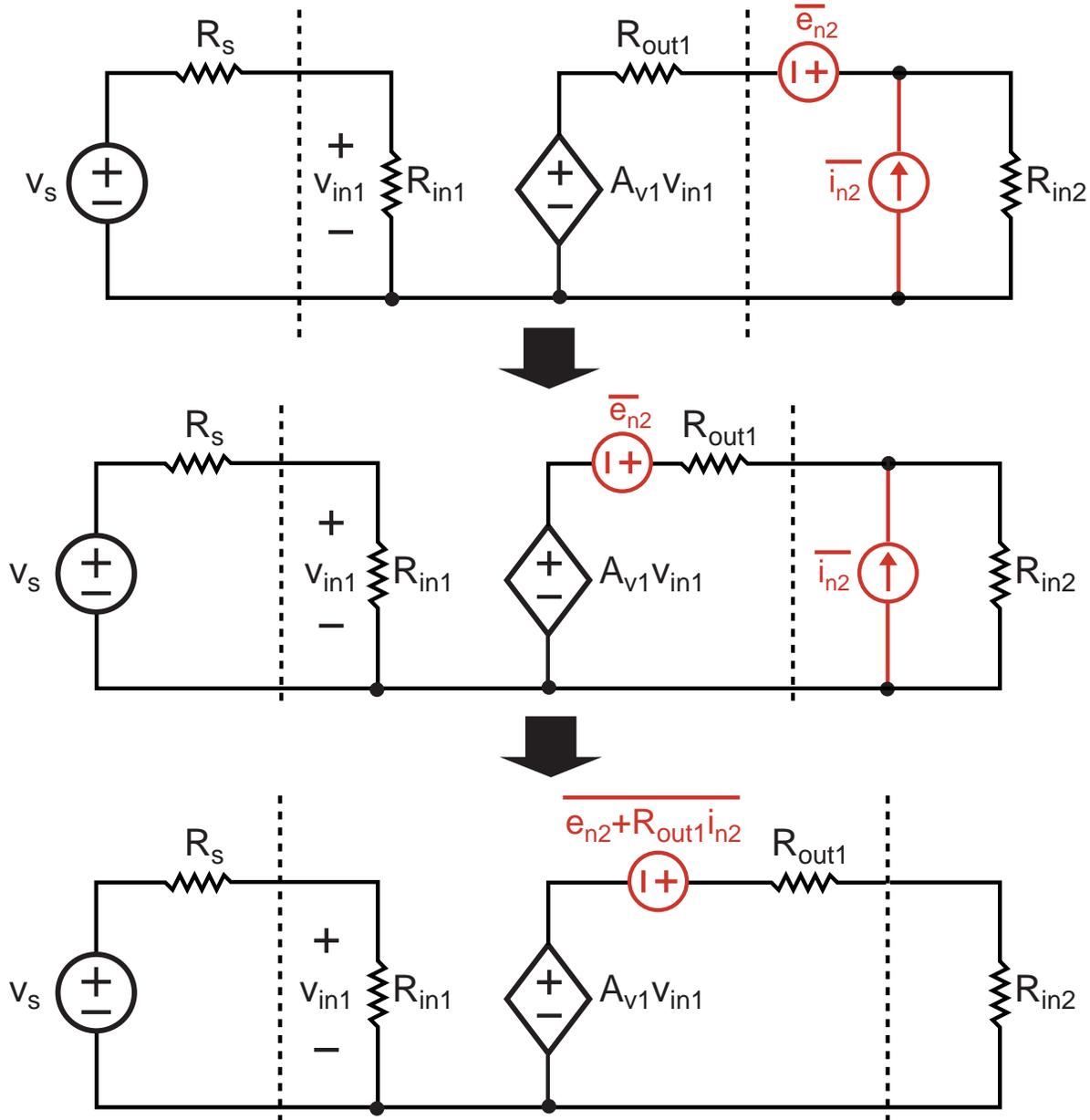
Method: Refer All Noise to Input of First Stage



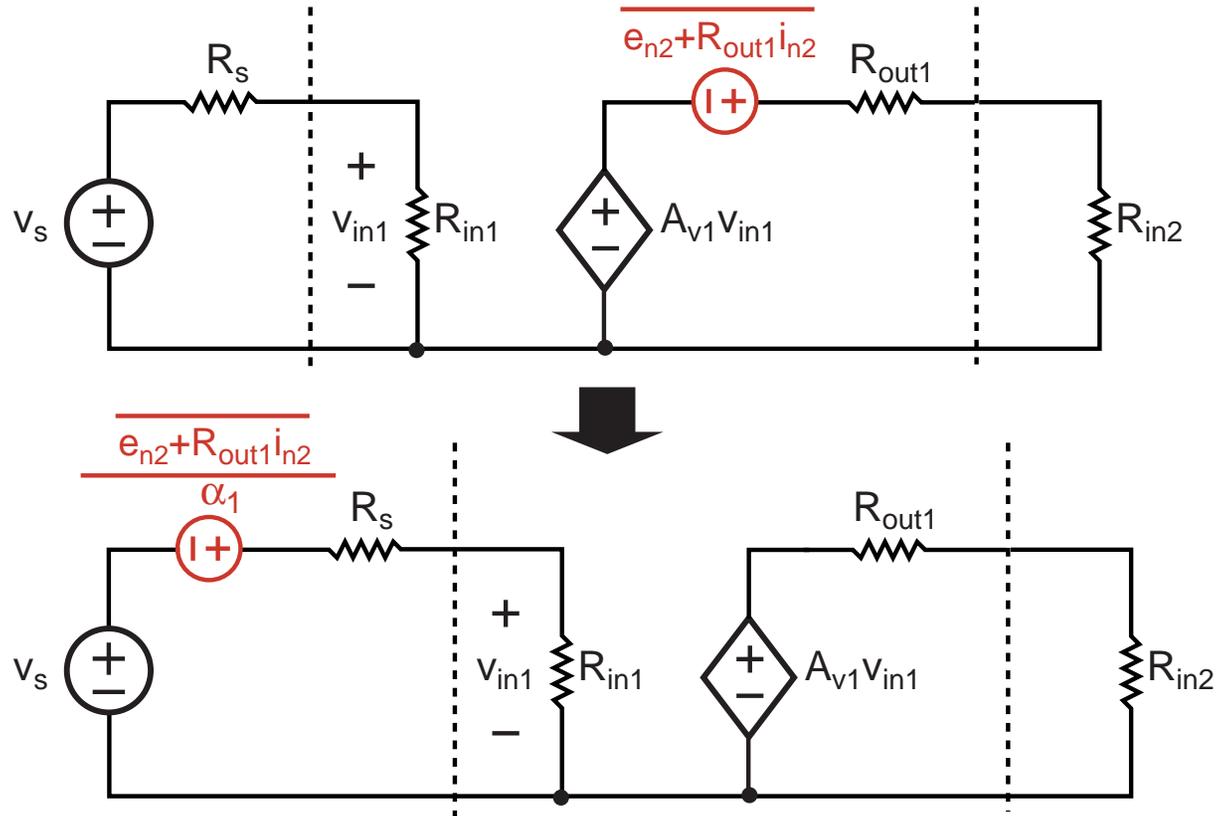
Model for referring stage 2 noise to input of stage 1



Step 1: Create an Equivalent Noise Voltage Source



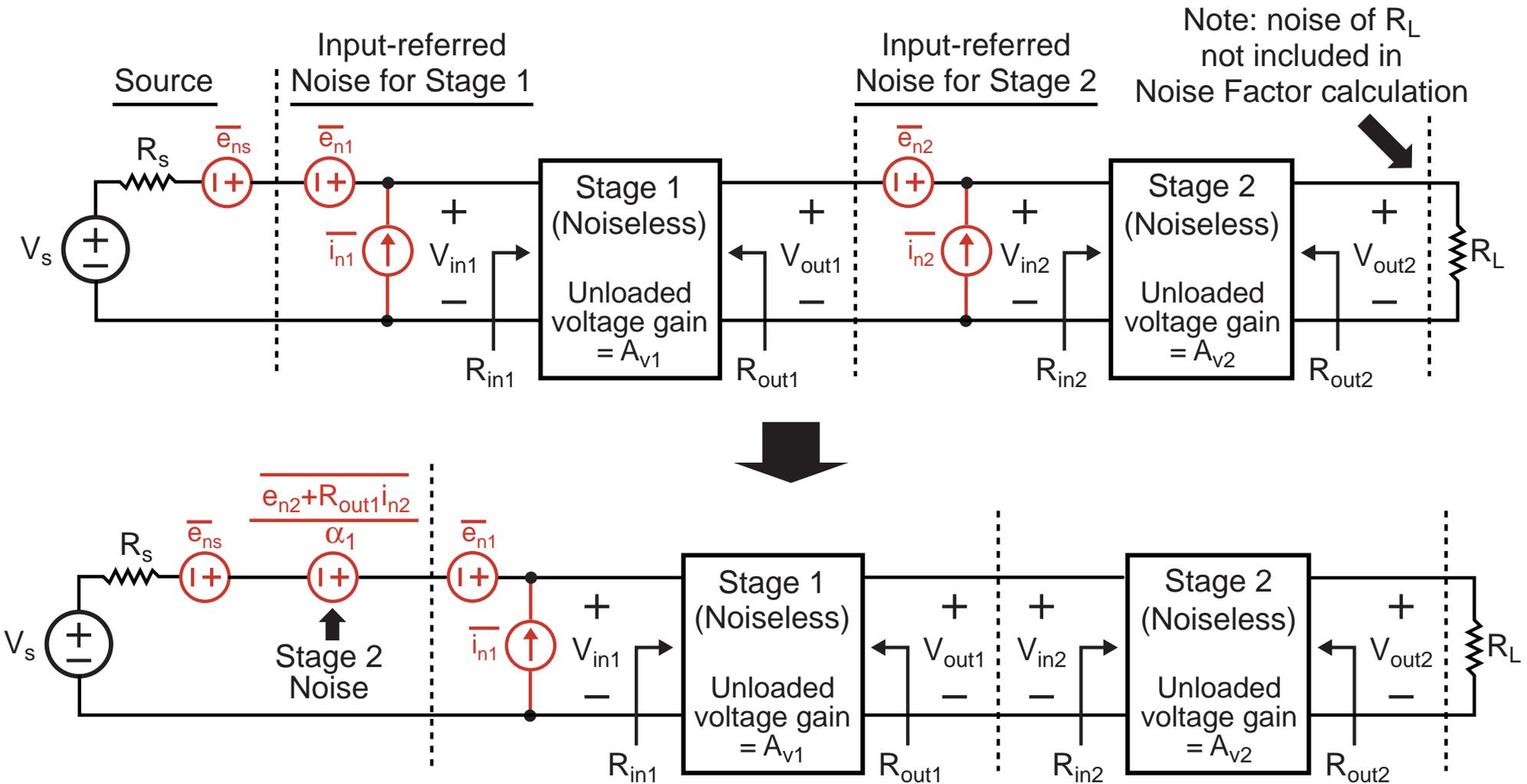
Step 2: Input Refer Voltage Noise Source



- Scaling factor α_1 is a function of unloaded gain, A_v , and input voltage divider

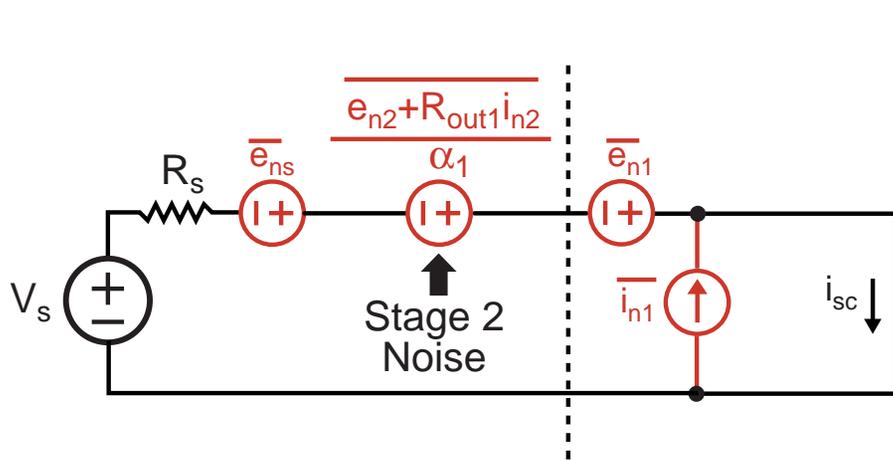
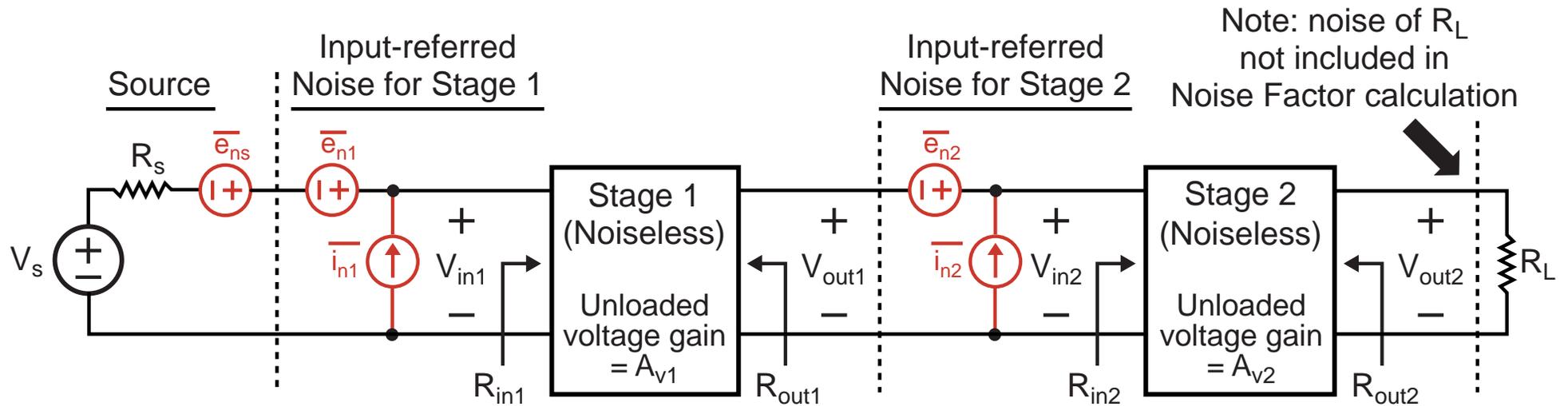
$$\alpha_1 = \frac{R_{in}}{R_s + R_{in}} A_v$$

Input Referral of Noise to First Stage



■ Where
$$\alpha_1 = \frac{R_{in}}{R_s + R_{in}} A_{v1}$$

Noise Factor Calculation

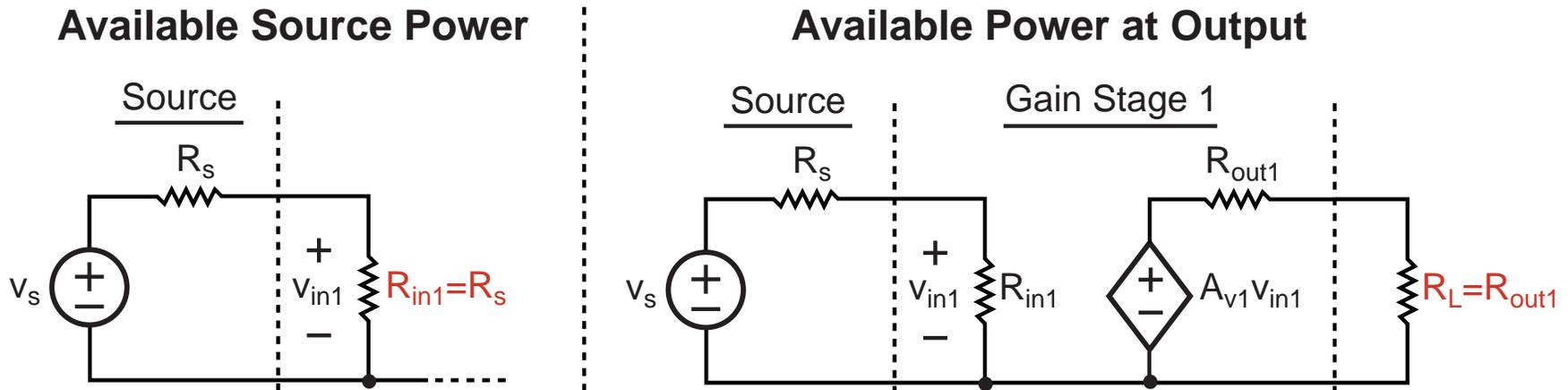


$$\Rightarrow NF = \frac{\left(\overline{e_{ns}^2} + \overline{(e_{n2} + R_{out1}i_{n2})^2} / \alpha_1^2 + \overline{(e_{n1} + R_s i_{n1})^2} \right) / R_s^2}{\overline{e_{ns}^2} / R_s^2}$$

Alternate Noise Factor Expression

$$\begin{aligned}
 \text{NF} &= \frac{\left(\overline{e_{ns}^2} + \overline{(e_{n2} + R_{out1}i_{n2})^2} / \alpha_1^2 + \overline{(e_{n1} + R_s i_{n1})^2} \right) / R_s^2}{\overline{e_{ns}^2} / R_s^2} \\
 &= \frac{\overline{e_{ns}^2} + \overline{(e_{n1} + R_s i_{n1})^2} + \overline{(e_{n2} + R_{out1}i_{n2})^2} / \alpha_1^2}{\overline{e_{ns}^2}} \\
 &= 1 + \frac{\overline{(e_{n1} + R_s i_{n1})^2}}{\overline{e_{ns}^2}} + \frac{1}{\alpha_1^2} \frac{\overline{(e_{n2} + R_{out1}i_{n2})^2}}{\overline{e_{ns}^2}} \\
 &= 1 + \frac{\overline{(e_{n1} + R_s i_{n1})^2}}{4kTR_s} + \frac{1}{\alpha_1^2} \frac{R_{out1}}{R_s} \frac{\overline{(e_{n2} + R_{out1}i_{n2})^2}}{4kTR_{out1}} \\
 &= 1 + (\text{NF}_1 - 1) + \frac{1}{\alpha_1^2} \frac{R_{out1}}{R_s} (\text{NF}_2 - 1) \\
 &= 1 + (\text{NF}_1 - 1) + \frac{(\text{NF}_2 - 1)}{A_{p1}}, \quad A_{p1} = \alpha_1^2 \frac{R_s}{R_{out1}}
 \end{aligned}$$

Define “Available Power Gain”



- Available power gain for stage 1 defined as

$$A_{p1} = \frac{\text{Available power at output (matched load)}}{\text{Available source power (matched load)}}$$

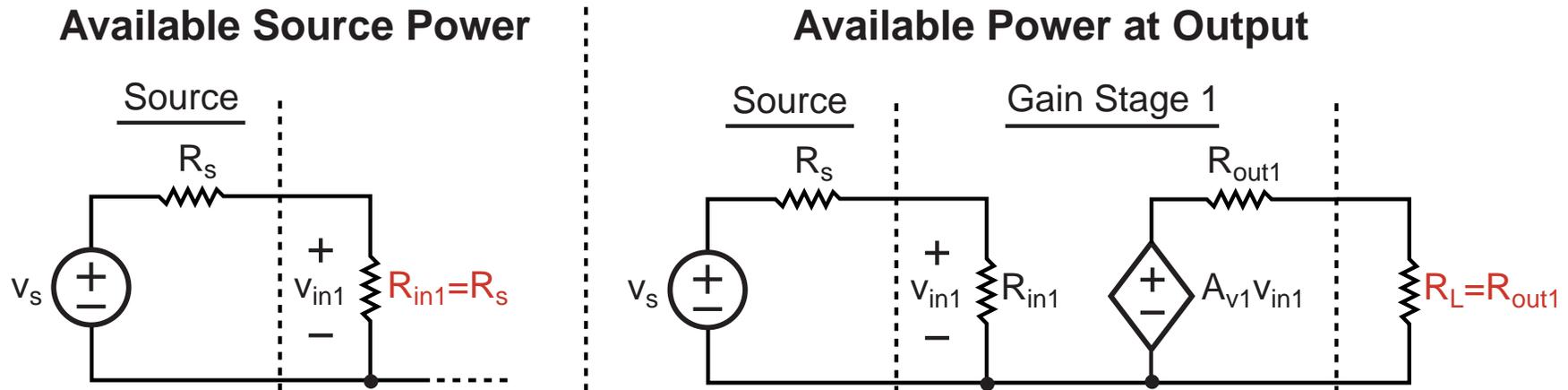
- Available power at output

$$\left(v_s \frac{R_{in1}}{R_s + R_{in1}} A_{v1} \frac{R_{out1}}{R_{out1} + R_{out1}} \right)^2 / R_{out1} = \frac{v_s^2 \alpha_1^2}{4R_{out1}}$$

- Available source power

$$\left(v_s \frac{R_s}{R_s + R_s} \right)^2 / R_s = \frac{v_s^2}{4R_s}$$

Available Power Gain Versus Loaded Voltage Gain



■ Available power gain for stage 1

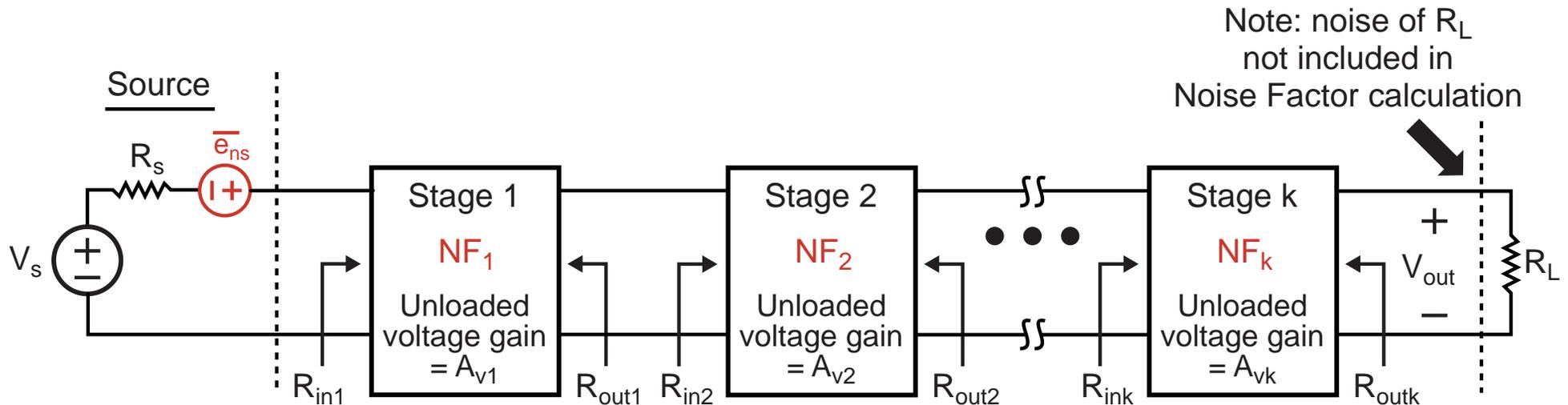
$$A_{p1} = \frac{v_s^2 \alpha_1^2}{4 R_{out1} v_s^2} \frac{4 R_s}{v_s^2} = \alpha_1^2 \frac{R_s}{R_{out1}} \quad \text{where} \quad \alpha_1 = \frac{R_{in1}}{R_s + R_{in1}} A_{v1}$$

■ If $R_{in1} = R_{out1} = R_s$

$$\alpha_1 = \frac{1}{2} A_{v1} \Rightarrow A_{p1} = \frac{1}{4} A_{v1}^2 = A_{v1_l}^2$$

— Where A_{v1_l} is defined as the “loaded gain” of stage 1

Final Expressions for Cascaded Noise Factor Calculation



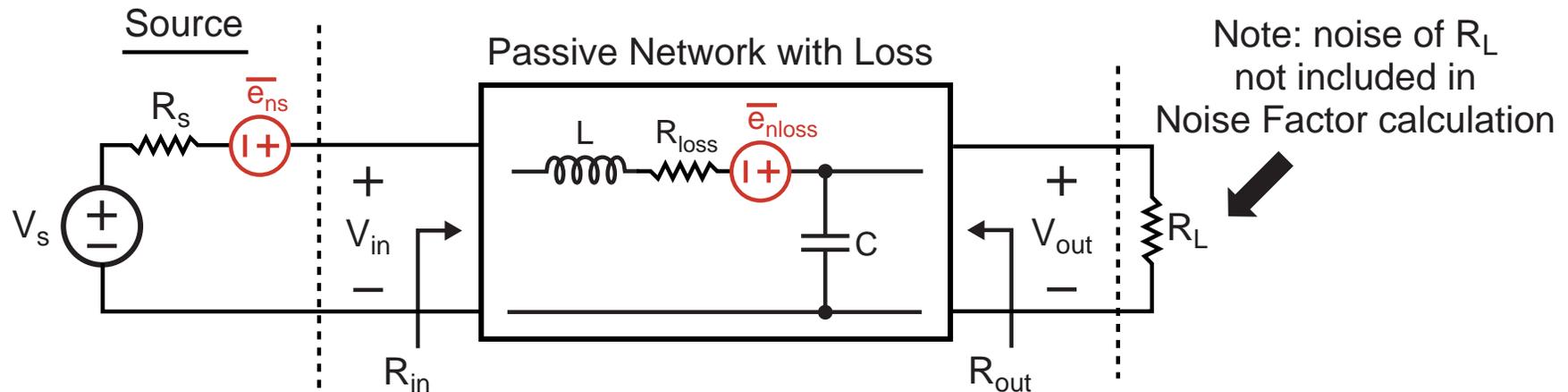
- Overall Noise Factor (general expression)

$$NF = 1 + (NF_1 - 1) + \frac{(NF_2 - 1)}{A_{p1}} + \dots + \frac{(NF_k - 1)}{A_{p1} \cdots A_{pk}}$$

- Overall Noise Factor when all input and output impedances equal R_s :

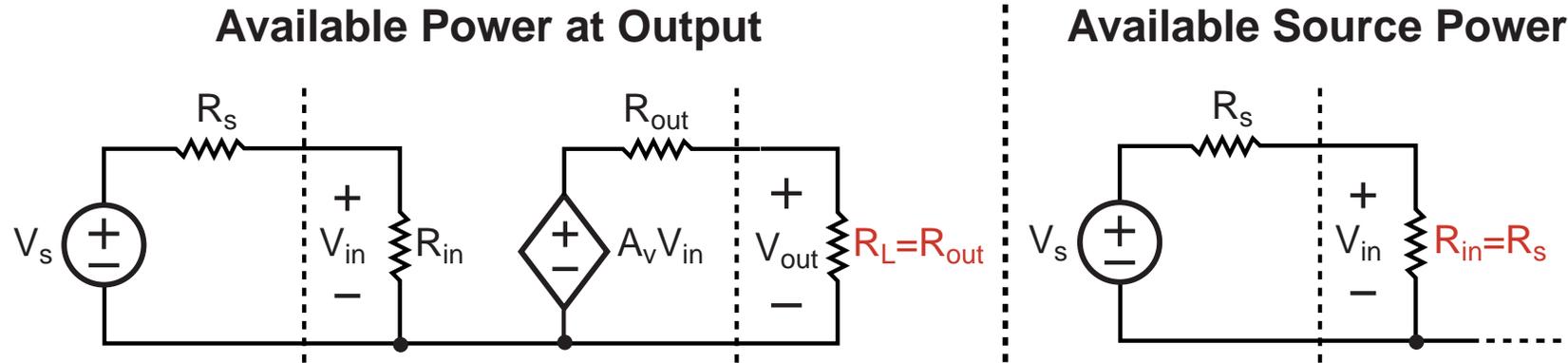
$$NF = 1 + (NF_1 - 1) + \frac{(NF_2 - 1)}{A_{v1-l}^2} + \dots + \frac{(NF_k - 1)}{A_{v1-l}^2 \cdots A_{vk-l}^2}$$

Calculation of Noise Factor for Lossy Passive Networks



- RF systems often employ passive filters for band select and channel select operations
 - Achieve high dynamic range and excellent selectivity
- Practical filters have loss
 - Can model as resistance in equivalent RLC network
 - Such resistance adds thermal noise, thereby lowering noise factor of receiver
- We would like to calculate noise factor contribution of lossy passive networks in a straightforward manner
 - See pages 46-48 of Razavi book

Define “Available Power Gain” For Passive Networks



- Available power at output

$$V_s^2 \left(\frac{R_{in}}{R_s + R_{in}} \right)^2 A_v^2 \left(\frac{R_{out}}{R_{out} + R_{out}} \right)^2 / R_{out} = \frac{V_s^2}{4R_{out}} \left(\frac{R_{in}}{R_s + R_{in}} \right)^2 A_v^2$$

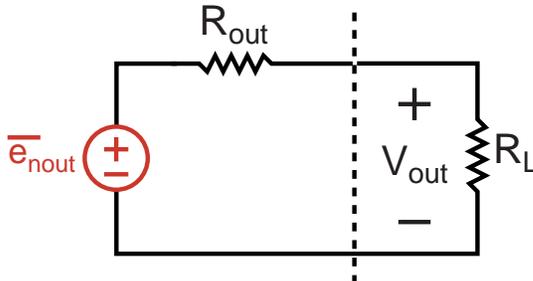
- Available source power

$$V_s^2 \left(\frac{R_s}{R_s + R_s} \right)^2 / R_{in} = \frac{V_s^2}{4R_s}$$

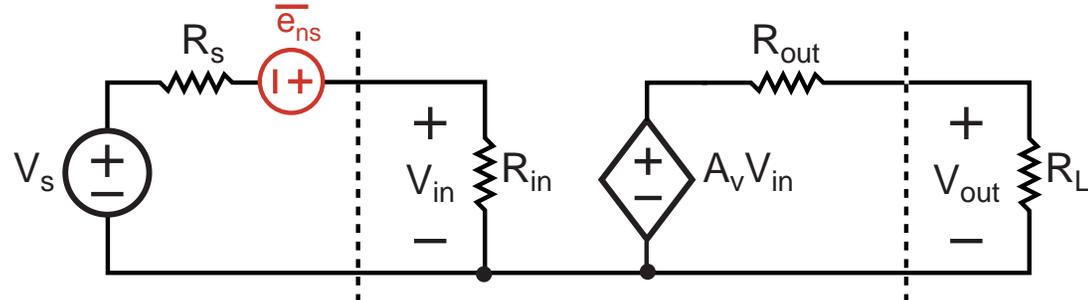
$$\Rightarrow A_p = \frac{R_s}{R_{out}} \left(\frac{R_{in}}{R_s + R_{in}} \right)^2 A_v^2 \quad (\leq 1 \text{ for passive networks})$$

Equivalent Noise Model and Resulting NF Calculation

Equivalent Model for Computing Total Noise



Equivalent Model for Computing Source Noise Contribution



- **Total noise at output**

$$\overline{e_{nout}^2} \left(\frac{R_L}{R_L + R_{out}} \right)^2 = 4kTR_{out} \left(\frac{R_L}{R_L + R_{out}} \right)^2$$

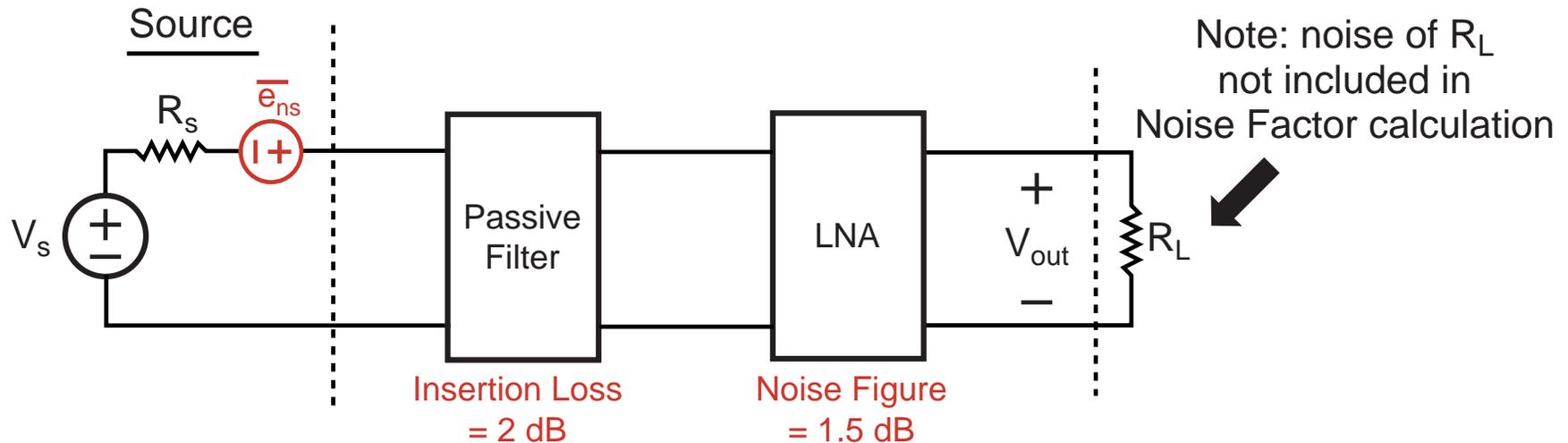
- **Noise due to source (referred to output)**

$$4kTR_s \left(\frac{R_{in}}{R_s + R_{in}} \right)^2 A_v^2 \left(\frac{R_L}{R_L + R_{out}} \right)^2$$

- **Noise factor**

$$NF = \frac{R_{out}}{R_s} \left(\frac{R_s + R_{in}}{R_{in}} \right)^2 \frac{1}{A_v^2} = \boxed{1/A_p}$$

Example: Impact of Cascading Passive Filter with LNA



■ Noise Factor calculation

$$NF = 1 + (NF_{filt} - 1) + \frac{NF_{LNA} - 1}{A_{p_filt}}$$

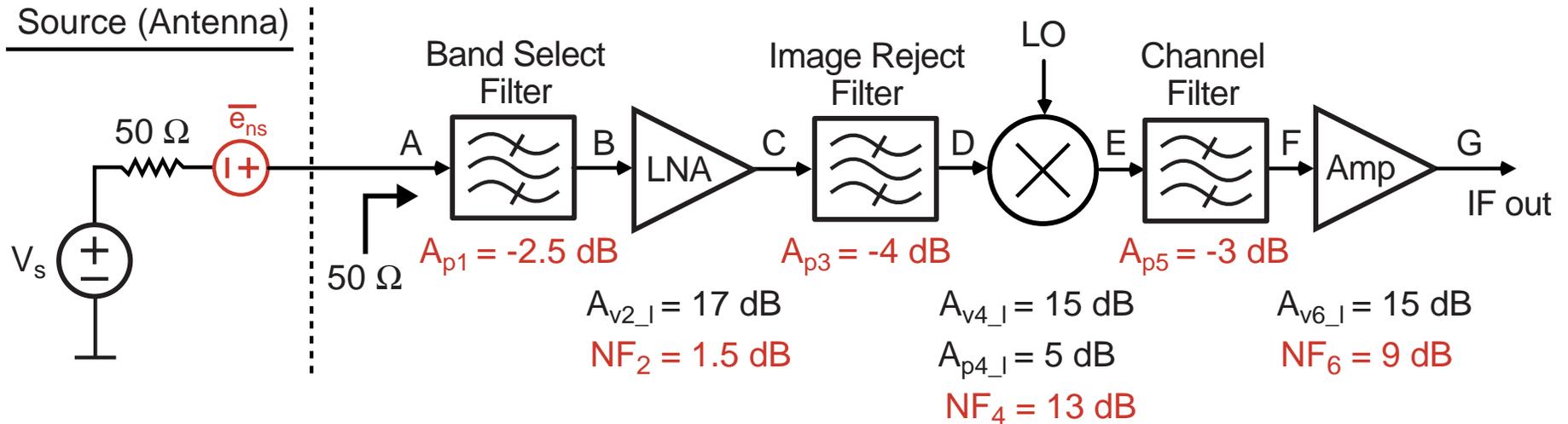
$$= 1 + (1/A_{p_filt} - 1) + \frac{NF_{LNA} - 1}{A_{p_filt}} = 1/A_{p_filt} \cdot NF_{LNA}$$

■ Noise Figure

$$10 \log(NF) = -10 \log(A_{p_filt}) + 10 \log(NF_{LNA})$$

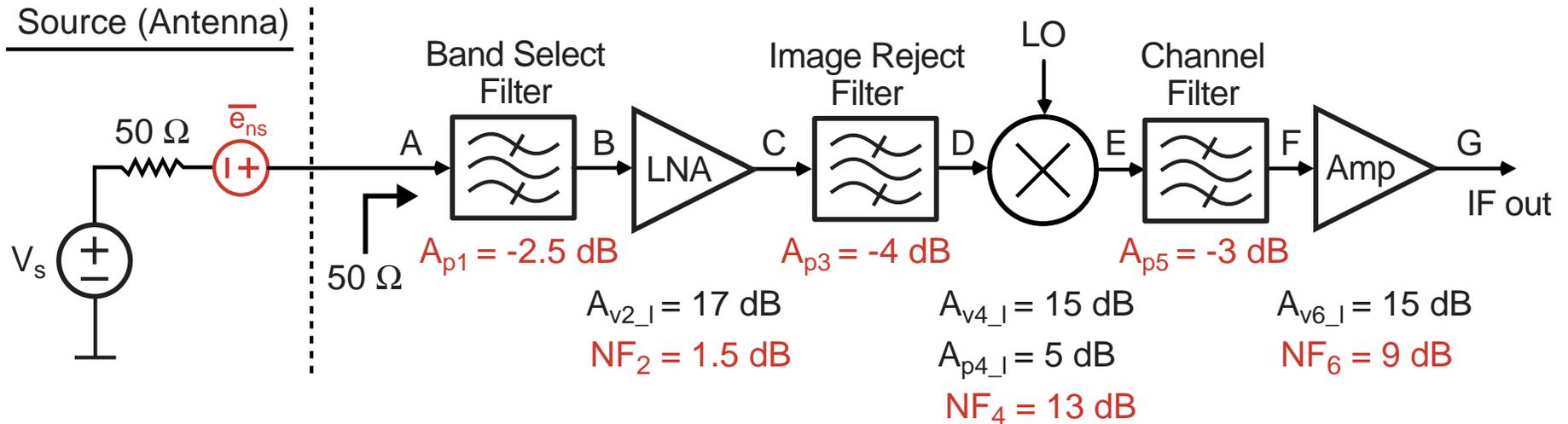
$$= 2 + 1.5 = 3.5 \text{ dB}$$

Example: Noise Factor Calculation for RF Receiver



- Ports A, B, C, and D are conjugate-matched for an impedance of 50 Ohms
 - Noise figure of LNA and mixer are specified for source impedances of 50 Ohms
- Ports E and F are conjugate-matched for an impedance of 500 Ohms
 - Noise figure of rightmost amplifier is specified for a source impedance of 500 Ohms

Methodology for Cascaded NF Calculation



- Perform Noise Figure calculations from right to left
- Calculation of cumulative Noise Factor at node k

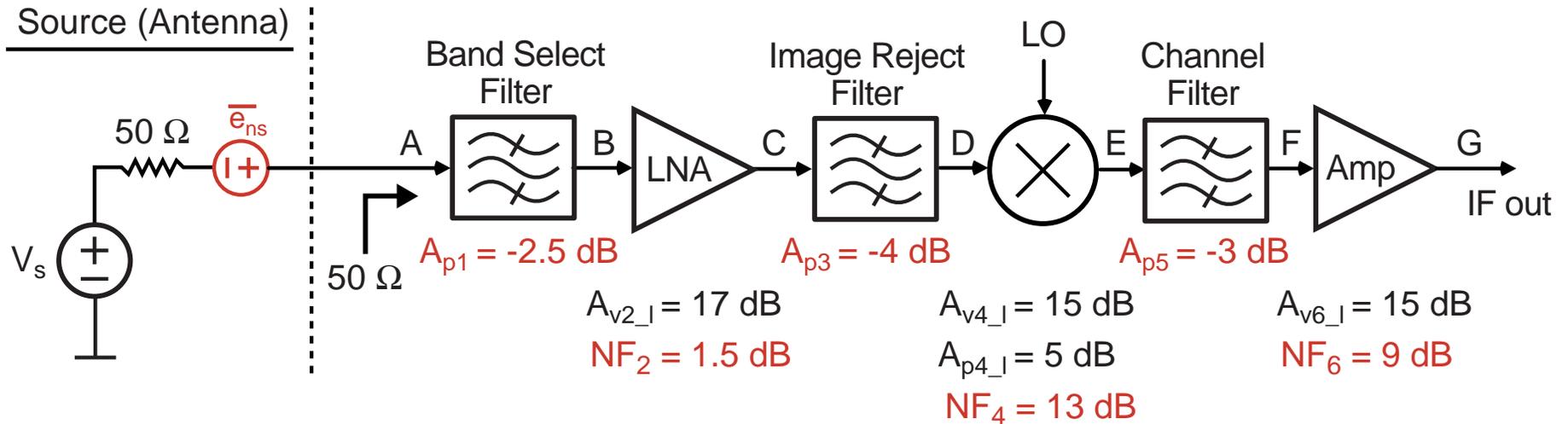
$$NF_cum_k = NF_k + \frac{(NF_{k+1} - 1)}{A_{pk}}$$

- If source and load impedances are equal

$$NF_cum_k = NF_k + \frac{(NF_{k+1} - 1)}{A_{vk}^2}$$

- True for all blocks except mixer above

Cumulative Noise Factor Calculations



$$NF_E = 10^{(3+9)/10} = 15.85 \quad (12 \text{ dB})$$

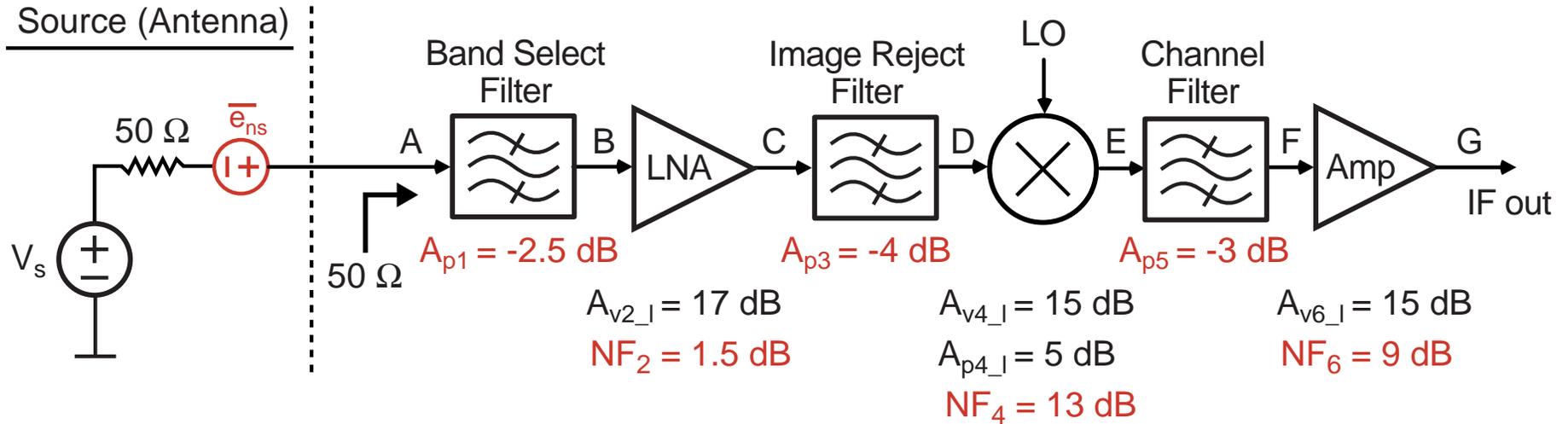
$$NF_D = 10^{(13)/10} + (15.85 - 1) / 10^{(5/10)} = 24.65 \quad (13.9 \text{ dB})$$

$$NF_C = 10^{(13.9+4)/10} = 61.7 \quad (17.9 \text{ dB})$$

$$NF_B = 10^{1.5/10} + (61.7 - 1) / 10^{17/10} = 2.62 \quad (4.2 \text{ dB})$$

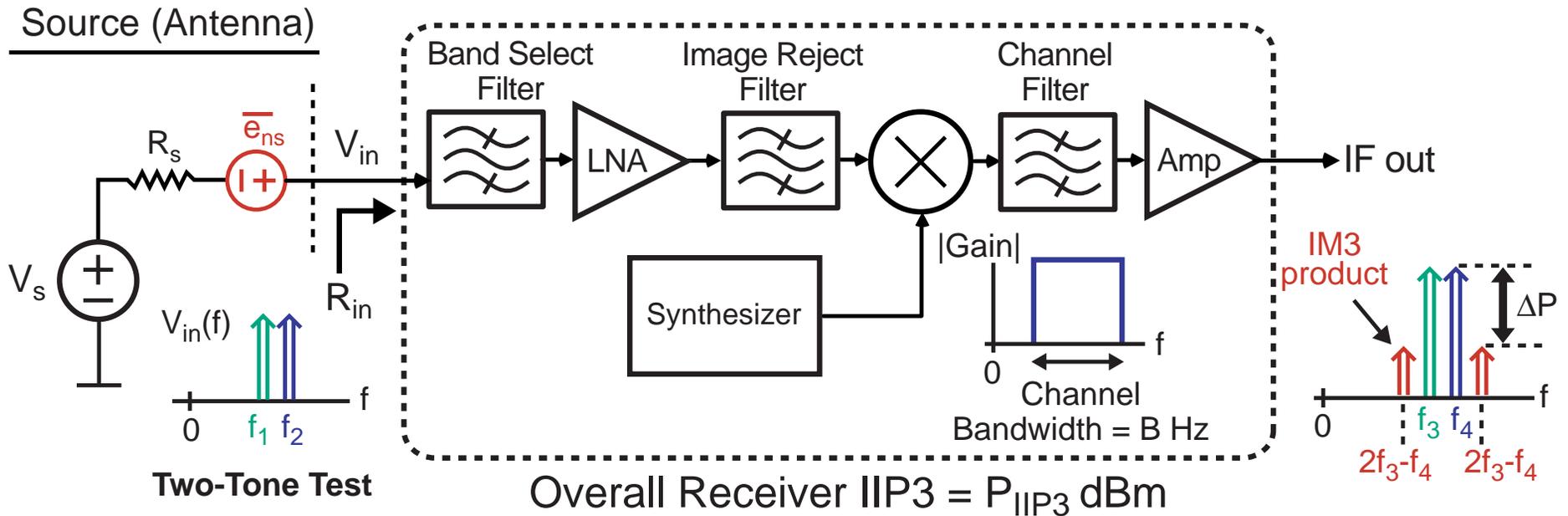
$$NF_A = 10^{(2.5+4.2)/10} = 4.68 \quad (6.7 \text{ dB})$$

“Level Diagram” for Gain, NF Calculation



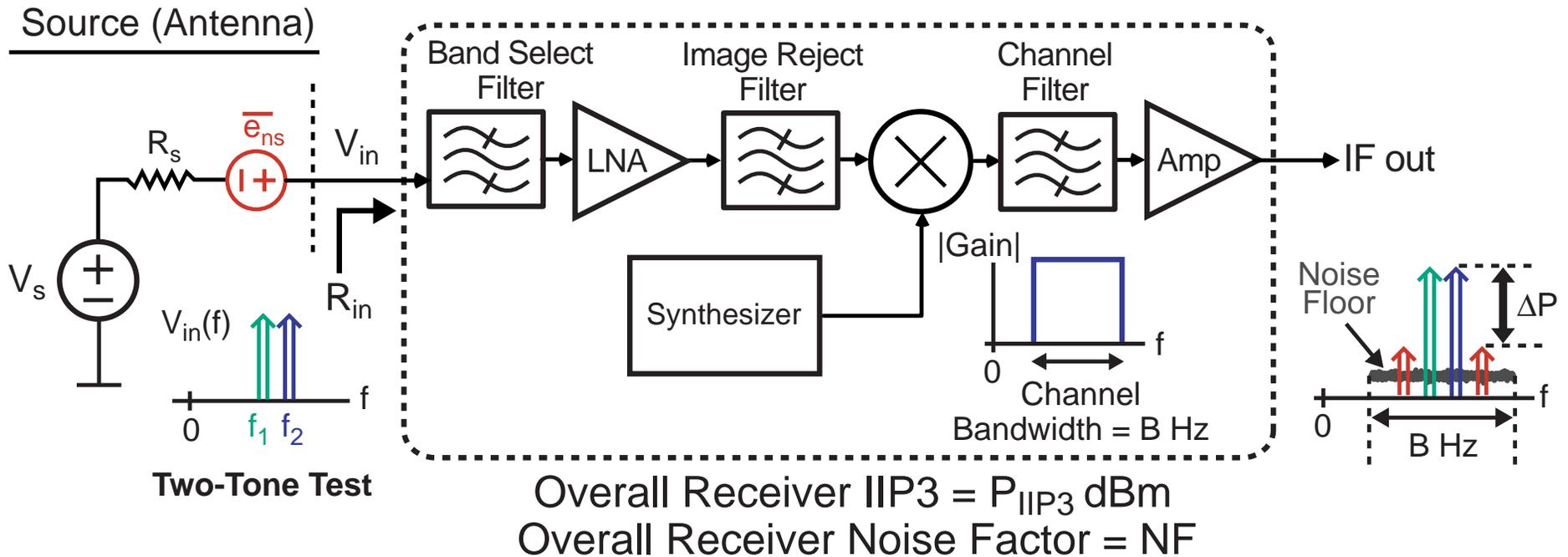
	A	B	C	D	E	F	G
Stage Gain (dB)							
Voltage (loaded)	-2.5	17	-4	15	-3	15	
Power	-2.5	17	-4	5	-3	15	
Cumulative Voltage Gain (dB)		-2.5	14.5	10.5	25.5	22.5	37.5
Stage NF (dB)	2.5	1.5	4	13	3	9	
Cumulative NF (dB)	6.7	4.2	17.9	13.9	12	9	

The Issue of Receiver Nonlinearity



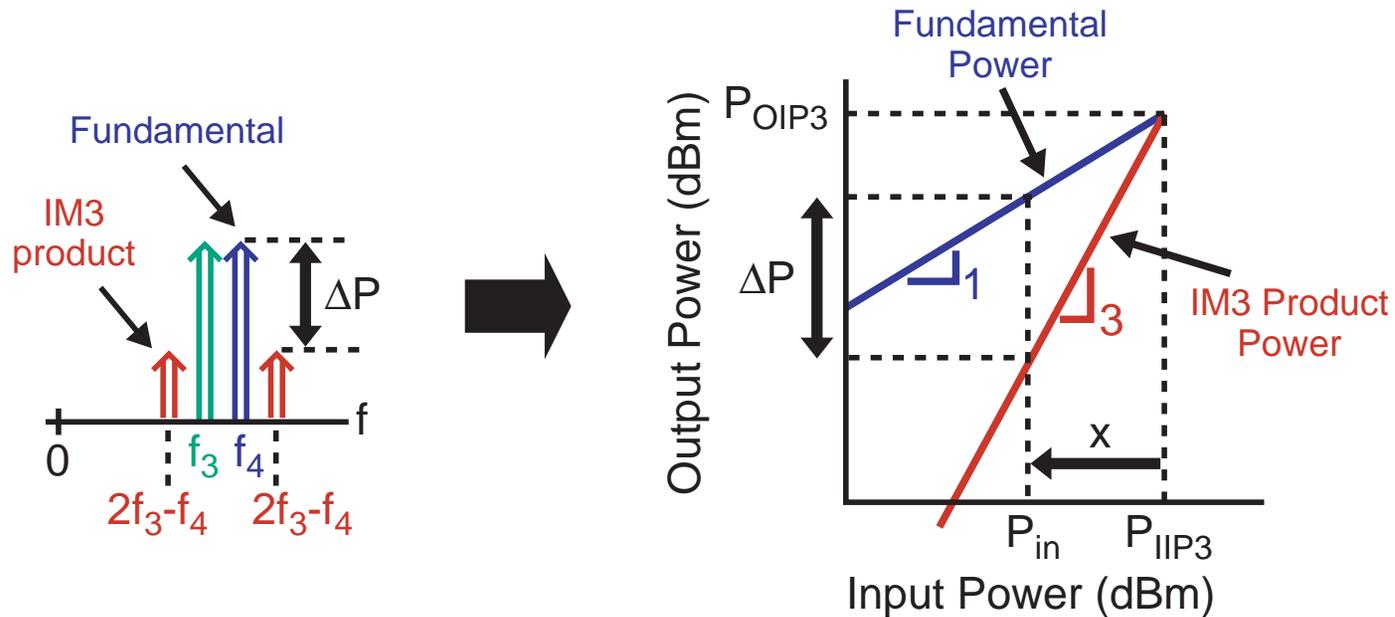
- Lower limit of input power into receiver is limited by sensitivity (i.e., required SNR, Noise Figure, etc.)
- Upper limit of input power into receiver is determined by nonlinear characteristics of receiver
 - High input power will lead to distortion that reduces SNR (even in the absence of blockers)
 - Nonlinear behavior often characterized by IIP3 performance of receiver

Receiver Dynamic Range



- **Defined as difference (in dB) between max and min input power levels to receiver**
 - **Min input power level set by receiver sensitivity**
 - **Max input power set by nonlinear characteristics of receiver**
 - Often defined as max input power for which third order IM products do not exceed the noise floor in a two tone test

A Key IIP3 Expression



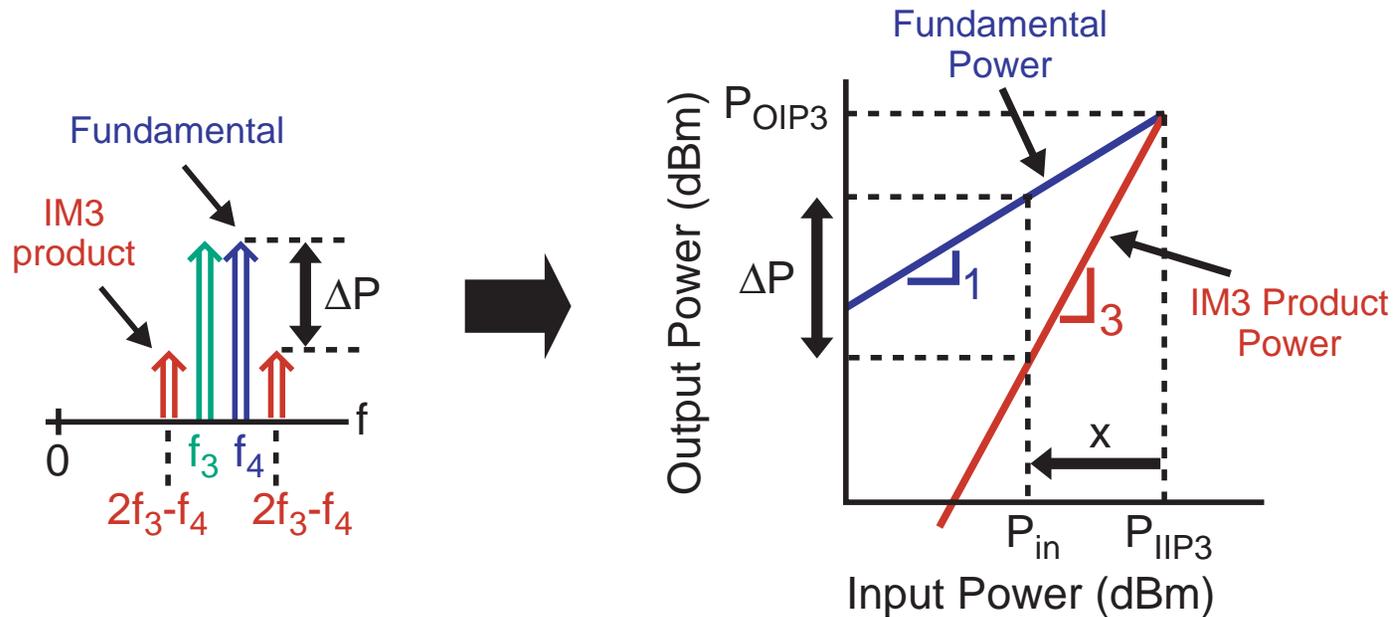
- By inspection of the right figure

$$P_{IIP3} = P_{in} + x \quad \Delta P = 3x - x = 2x$$

- Combining the above expressions:

$$\Rightarrow P_{IIP3} = P_{in} + \frac{\Delta P}{2} = P_{in} + \frac{P_{out} - P_{IM3,out}}{2}$$

Refer All Signals to Input in Previous IIP3 Expression



- **Difference between fundamental and IM3 products, ΔP , is the same (in dB) when referred to input of amplifier**
 - Both are scaled by the inverse of the amplifier gain

$$\Rightarrow P_{IIP3} = P_{in} + \frac{\Delta P}{2} = P_{in} + \frac{P_{in} - P_{IM3,in}}{2}$$

- **Applying algebra:**

$$P_{in} = \frac{2P_{IIP3} + P_{IM3,in}}{3}$$

Calculation of Spurious Free Dynamic Range (SFDR)

- **Key expressions:**

- Minimum P_{in} (dBm) set by SNR_{min} and noise floor

$$P_{in,min} = F + SNR_{out,min}$$

- Where F is the input referred noise floor of the receiver

$$F = -174 + 10 \log(B) + dB(NF)$$

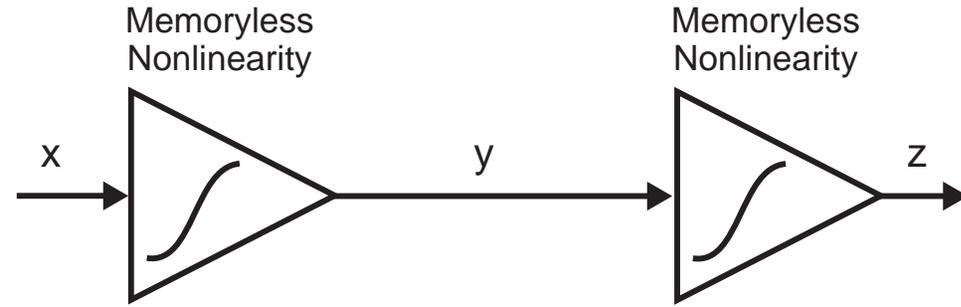
- Max P_{in} (dBm) occurs when IM3 products = noise floor

$$P_{in,max} = \frac{2P_{IIP3} + P_{IM3,in,max}}{3} \Rightarrow P_{in,max} = \frac{2P_{IIP3} + F}{3}$$

- **Dynamic range: subtract min from max P_{in} (in dB)**

$$SFDR = \frac{2P_{IIP3} + F}{3} - (F + SNR_{out,min})$$

Calculation of Overall IIP3 for Cascaded Stages



- Assume nonlinearity of each stage characterized as

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$z(t) = \beta_1 y(t) + \beta_2 y^2(t) + \beta_3 y^3(t)$$

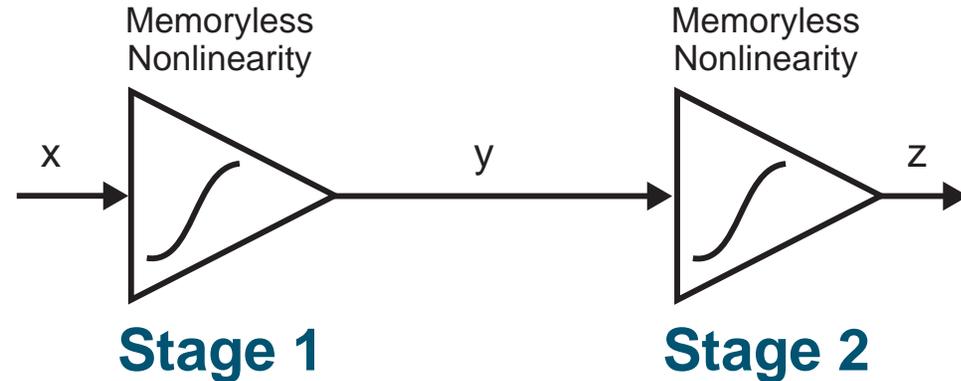
- Multiply nonlinearity expressions and focus on first and third order terms

$$z(t) = \alpha_1 \beta_1 x(t) + (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) x^3(t) + \dots$$

- Resulting IIP3 expression

$$A_{IP3} = \sqrt{\frac{4}{3} \left| \frac{\alpha_1 \beta_1}{\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3} \right|}$$

Alternate Expression for Overall IIP3



- Worst case IIP3 estimate – take absolute values of terms

$$A_{IP3} \approx \sqrt{\frac{4 |\alpha_1 \beta_1|}{3 |\alpha_3 \beta_1| + |2 \alpha_1 \alpha_2 \beta_2| + |\alpha_1^3 \beta_3|}}$$

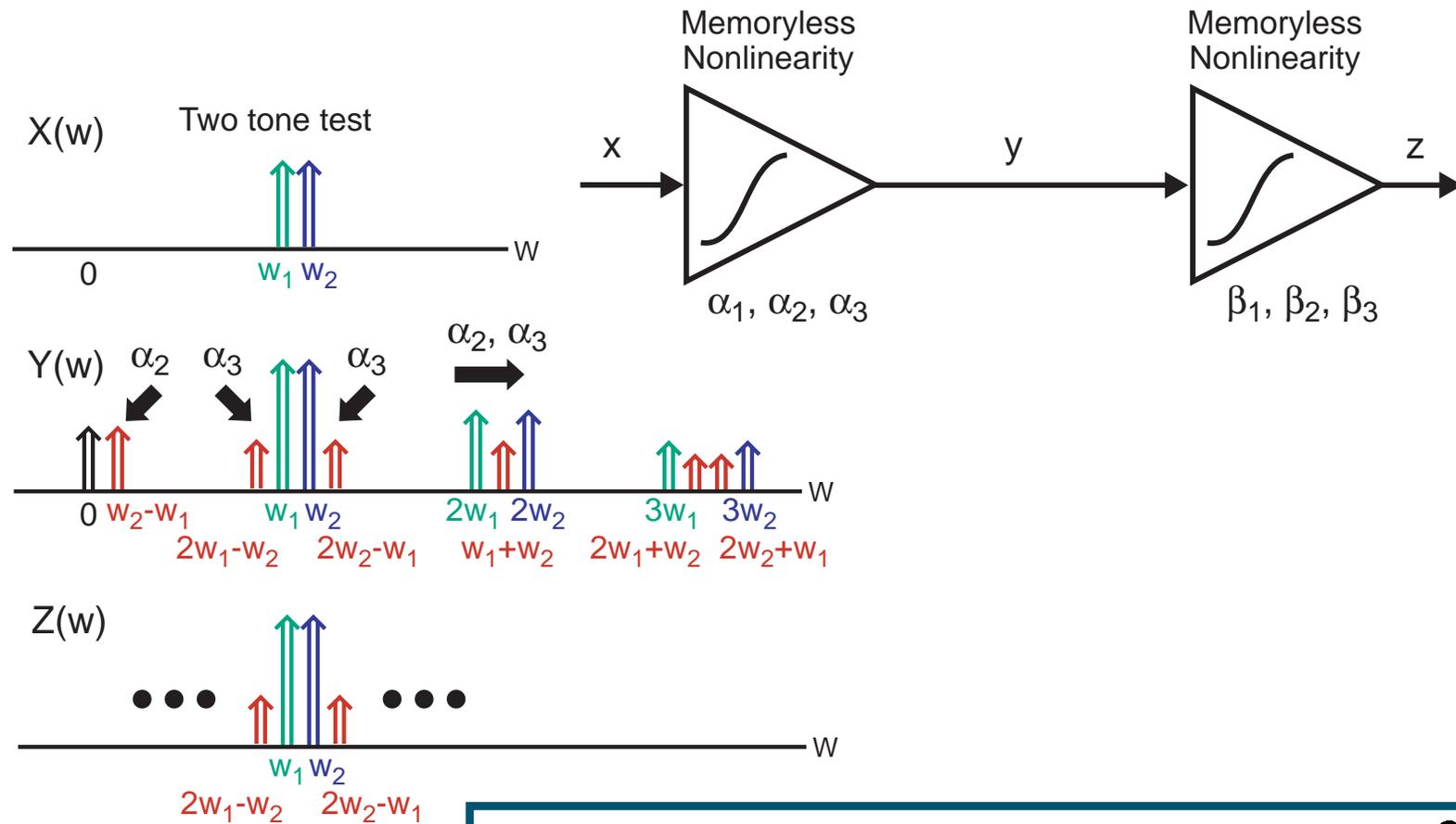
- Square and invert the above expression

$$\frac{1}{A_{IP3}^2} \approx \frac{3 |\alpha_3 \beta_1| + |2 \alpha_1 \alpha_2 \beta_2| + |\alpha_1^3 \beta_3|}{4 |\alpha_1 \beta_1|}$$

- Express formulation in terms of IIP3 of stage 1 and stage 2

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{3 |\alpha_2 \beta_2|}{2 |\beta_1|} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$

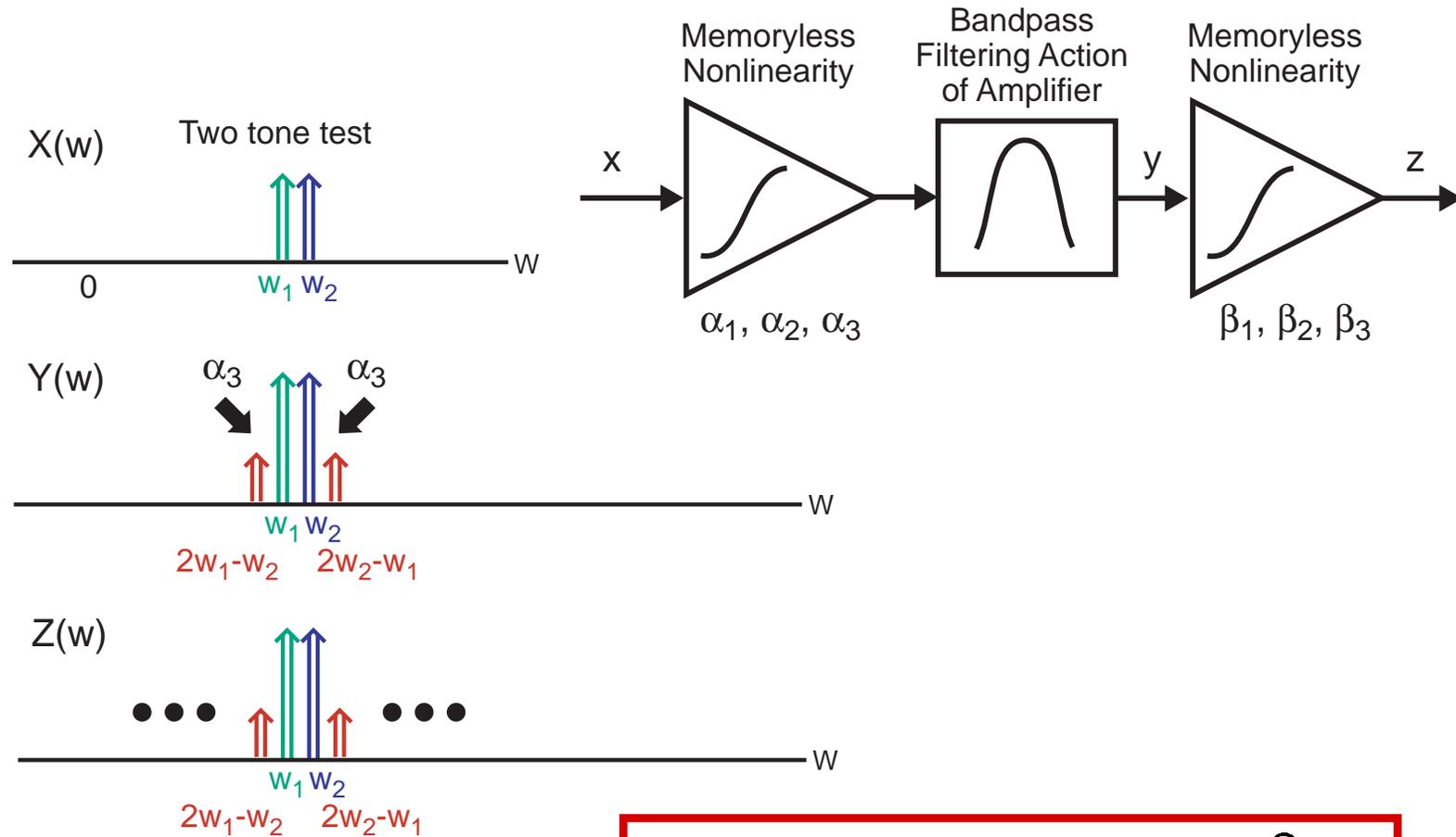
A Closer Look at Impact of Second Order Nonlinearity



$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{3|\alpha_2\beta_2|}{2|\beta_1|} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$

- Influence of α_2 of Stage 1 produces tones that are at frequencies far away from two tone input

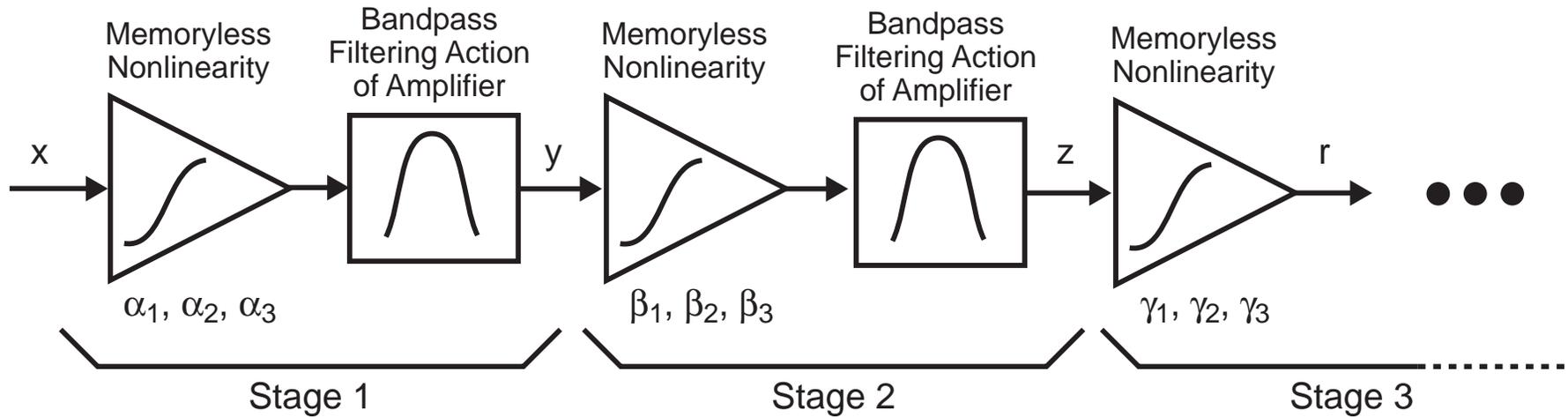
Impact of Having Narrowband Amplification



$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$

- Removal of outside frequencies dramatically simplifies overall IIP3 calculation

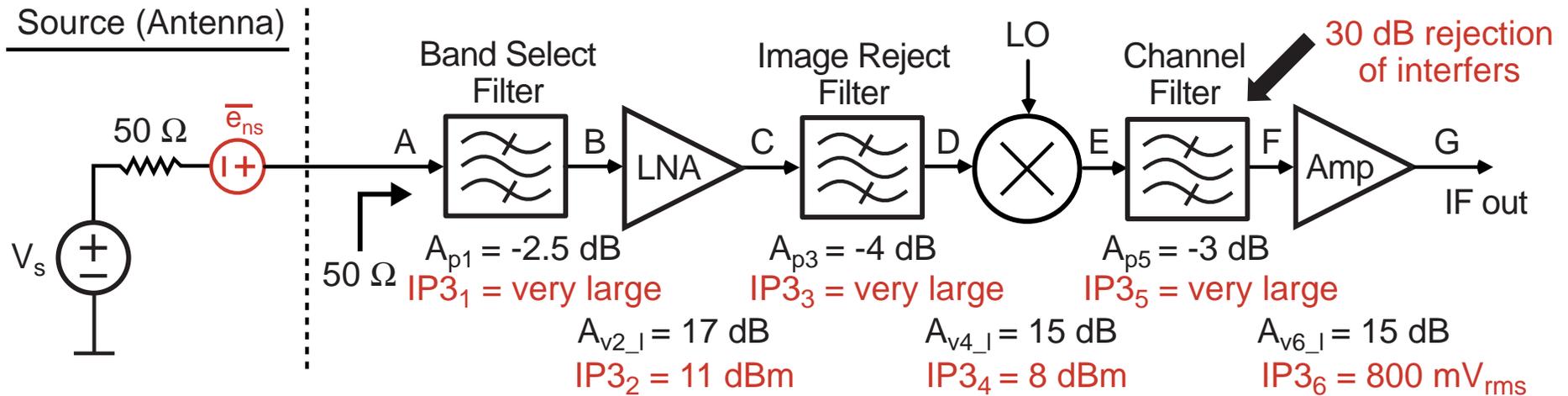
Cascaded $IIP3$ Calculation with Narrowband Stages



$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2 \beta_1^2}{A_{IP3,3}^2} + \dots$$

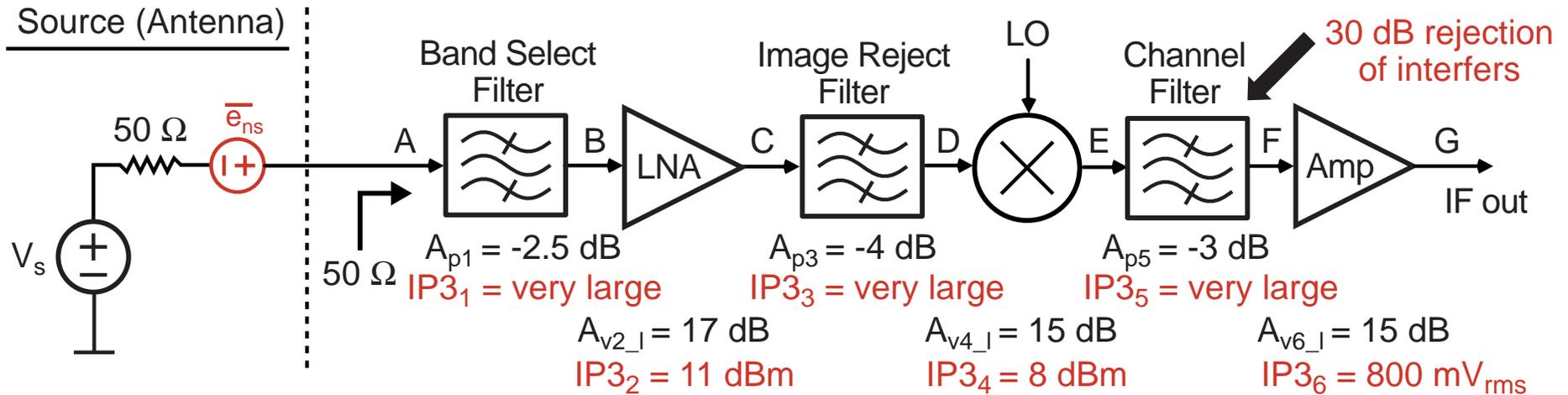
- Note that α_1 and β_1 correspond to the loaded voltage gain values for Stage 1 and 2, respectively

Example: IIP3 Calculation for RF Receiver



- Ports A, B, C, and D are conjugate-matched for an impedance of 50 Ohms
 - IIP3 of LNA and mixer are specified for source impedances of 50 Ohms
- Ports E and F and conjugate-matched for an impedance of 500 Ohms
 - IIP3 of rightmost amplifier is specified for a source impedance of 500 Ohms

Key Formulas for IIP3 Calculation



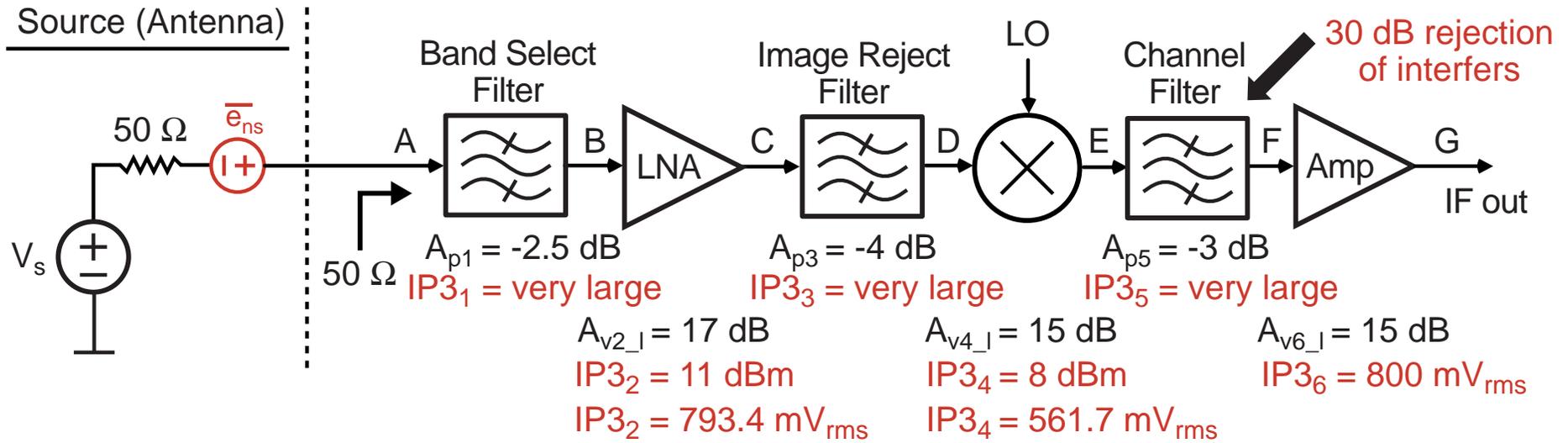
- Perform IIP3 calculations from right to left
- Calculation of cumulative IIP3 at node k (IIP3 in units of rms voltage)

$$IIP3_cum_k = 1 / \sqrt{1 / IIP3_k^2 + A_{vk_l}^2 / IIP3_{k-1}^2}$$

- Conversion from rms voltage to dBm

$$dBm = 10 \log(1e3 \cdot V_{rms}^2 / R)$$

Cumulative IIP3 Calculations



$$IIP3_E = 1/\sqrt{1/\infty + 10^{(-30/10)}/0.8^2} = 25.3 V_{rms}$$

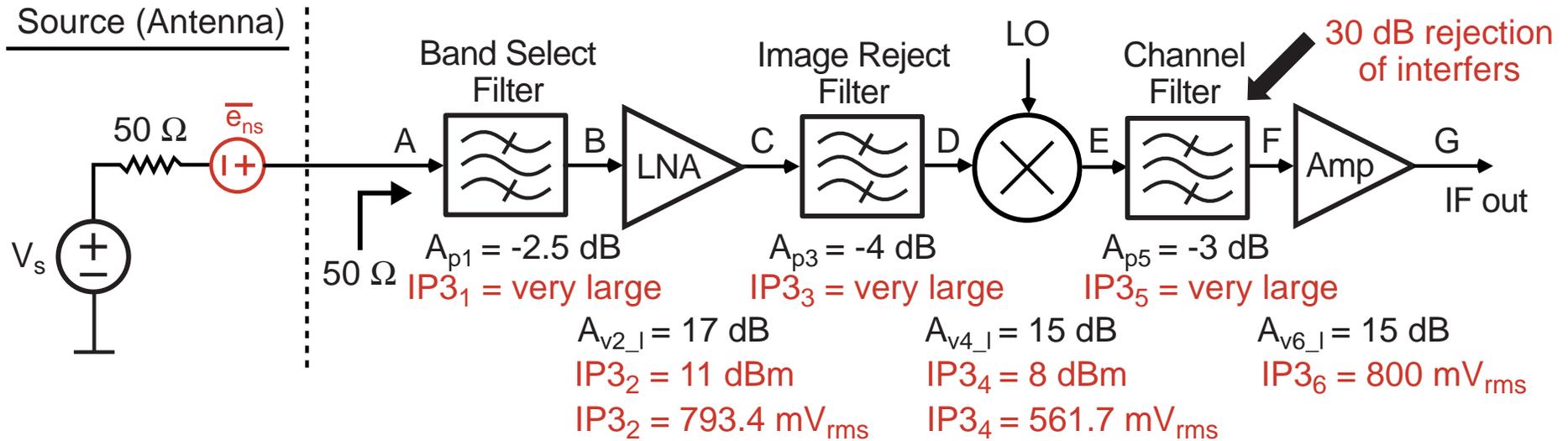
$$IIP3_D = 1/\sqrt{1/0.5617^2 + 10^{(15/10)}/25.3^2} = 557.4 mV_{rms}$$

$$IIP3_C = 1/\sqrt{1/\infty + 10^{(-4/10)}/0.5574^2} = 883.4 mV_{rms}$$

$$IIP3_B = 1/\sqrt{1/0.7934^2 + 10^{(17/10)}/0.8834^2} = 123.3 mV_{rms}$$

$$IIP3_A = 1/\sqrt{1/\infty + 10^{(-2.5/10)}/0.1233^2} = 164.4 mV_{rms}$$

“Level Diagram” for Gain, IIP3 Calculations



	A	B	C	D	E	F	G
Stage Gain (dB)							
Voltage (loaded)	-2.5	17	-4	15	-3	15	
Power	-2.5	17	-4	5	-3	15	
Cumulative Voltage Gain (dB)		-2.5	14.5	10.5	25.5	22.5	37.5
Stage IIP3							
Power (dBm)	Very large	11	Very large	8	Very large	1.07	
Voltage (mV _{rms})	Very large	793.4	Very large	561.7	Very large	800	
Cumulative IIP3							
Voltage (mV _{rms})	164.4	123.3	883.4	557.4	25.3e3	800	
Power (dBm)	-2.67	-5.17	11.93	7.93	91	1.07	

Final Comments on IIP3 and Dynamic Range

- **Calculations we have presented assume**
 - **Narrowband stages**
 - Influence of second order nonlinearity removed
 - **IM3 products are the most important in determining maximum input power**
- **Practical issues**
 - **Narrowband operation cannot always be assumed**
 - **Direct conversion architectures are also sensitive to IM2 products (i.e., second order distortion)**
 - **Filtering action of channel filter will not reduce in-band IM3 components of blockers (as assumed in the previous example in node E calculation)**

Must perform simulations to accurately characterize IIP3 (and IIP2) and dynamic range of RF receiver