Short Course On
Phase-Locked Loops and Their Applications
Day 2, AM Lecture

Basic Building Blocks
Voltage-Controlled Oscillators

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### Design Issues
- **Tuning Range** – need to cover all frequency channels
- **Noise** – impacts receiver blocking and sensitivity performance
- **Power** – want low power dissipation
- **Isolation** – want to minimize noise pathways into VCO
- **Sensitivity to process/temp variations** – need to make it manufacturable in high volume
**VCO Design For High Speed Data Links**

- **Design Issues**
  - Same as wireless, but:
    - Required noise performance is often less stringent
    - Tuning range is often narrower
Outline of Talk

- Common oscillator implementations
- Barkhausen’s criterion of oscillation
- One-port view of resonance based oscillators
  - Impedance transformation
  - Negative feedback topologies
- Voltage controlled oscillators
Popular VCO Structures

- LC Oscillator: low phase noise, large area
- Ring Oscillator: easy to integrate, higher phase noise
Barkhausen’s Criteria for Oscillation

- **Closed loop transfer function**
  
  \[ G(jw) = \frac{Y(jw)}{X(jw)} = \frac{H(jw)}{1 - H(jw)} \]

- **Self-sustaining oscillation at frequency** \( w_o \) **if**
  
  \[ H(jw_o) = 1 \]

  - Amounts to two conditions:
    - Gain = 1 at frequency \( w_o \)
    - Phase = \( n \times 360 \) degrees \((n = 0, 1, 2, \ldots)\) at frequency \( w_o \)
Example 1: Ring Oscillator

- Gain is set to 1 by saturating characteristic of inverters
- Phase equals 360 degrees at frequency of oscillation
  - Assume N stages each with phase shift $\Delta \Phi$
    \[ 2N \Delta \Phi = 360^\circ \quad \Rightarrow \quad \Delta \Phi = \frac{180^\circ}{N} \]
  - Alternately, N stages with delay $\Delta t$
    \[ 2N \Delta t = T \quad \Rightarrow \quad \Delta t = \frac{T/2}{N} \]
Further Info on Ring Oscillators

- Due to their relatively poor phase noise performance, ring oscillators are rarely used in RF systems
  - They are used quite often in high speed data links, though
- We will focus on LC oscillators in this lecture
- Some useful info on CMOS ring oscillators
  - Todd Weigandt’s PhD thesis – http://kabuki.eecs.berkeley.edu/~weigandt/
Example 2: Resonator-Based Oscillator

Barkhausen Criteria for oscillation at frequency $w_0$:

$$G_m Z(jw_0) = 1$$

- Assuming $G_m$ is purely real, $Z(jw_0)$ must also be purely real
For parallel resonator at resonance
- Looks like resistor (i.e., purely real) at resonance
  - Phase condition is satisfied
  - Magnitude condition achieved by setting $G_m R_p = 1$
Impact of Different $G_m$ Values

- Root locus plot allows us to view closed loop pole locations as a function of open loop poles/zero and open loop gain ($G_m R_p$)
  - As gain ($G_m R_p$) increases, closed loop poles move into right half S-plane
Impact of Setting $G_m$ too low

- Closed loop poles end up in the left half S-plane
  - Underdamped response occurs
    - Oscillation dies out
Impact of Setting $G_m$ too High

- Closed loop poles end up in the right half S-plane
  - Unstable response occurs
    - Waveform blows up!
Setting $G_m$ To Just the Right Value

- Closed loop poles end up on jw axis
  - Oscillation maintained
- Issue – $G_m R_p$ needs to exactly equal 1
  - How do we achieve this in practice?
Amplitude Feedback Loop

- One thought is to detect oscillator amplitude, and then adjust $G_m$ so that it equals a desired value
  - By using feedback, we can precisely achieve $G_m R_p = 1$

- Issues
  - Complex, requires power, and adds noise
Leveraging Amplifier Nonlinearity as Feedback

- Practical transconductance amplifiers have saturating characteristics
  - Harmonics created, but filtered out by resonator
  - Our interest is in the relationship between the input and the fundamental of the output
Leveraging Amplifier Nonlinearity as Feedback

- As input amplitude is increased
  - Effective gain from input to fundamental of output drops
  - Amplitude feedback occurs! ($G_mR_p = 1$ in steady-state)
One-Port View of Resonator-Based Oscillators

- Convenient for intuitive analysis
- Here we seek to cancel out loss in tank with a negative resistance element
  - To achieve sustained oscillation, we must have

\[
\frac{1}{G_m} = R_p \quad \Rightarrow \quad G_m R_p = 1
\]
One-Port Modeling Requires Parallel RLC Network

- Since VCO operates over a very narrow band of frequencies, we can always do series to parallel transformations to achieve a parallel network for analysis.

  - Warning – in practice, RLC networks can have secondary (or more) resonant frequencies, which cause undesirable behavior.
    - Equivalent parallel network masks this problem in hand analysis
    - Simulation will reveal the problem
Understanding Narrowband Impedance Transformation

Series Resonant Circuit

\[ Z_{in} = \frac{1}{j\omega C_s} + j\omega L_s + R_s \]

\[ = R_s \quad \text{for} \quad w = \frac{1}{\sqrt{L_s C_s}} = w_o \]

\[ Q = \frac{w_o L_s}{R_s} = \frac{1}{w_o C_s R_s} \]

Parallel Resonant Circuit

\[ Z_{in} = \frac{1}{j\omega C_p}||j\omega L_p||R_p \]

\[ = R_p \quad \text{for} \quad w = \frac{1}{\sqrt{L_p C_p}} = w_o \]

\[ Q = \frac{R_p}{w_o L_p} = w_o C_p R_p \]

- **Note:** resonance allows \( Z_{in} \) to be purely real despite the presence of reactive elements
Comparison of Series and Parallel RL Circuits

Series RL Circuit

\[ Z_{in} = jw_0L_s + R_s \]

\[ Q = \frac{w_0L_s}{R_s} \]

Parallel RL Circuit

\[ Z_{in} = jw_0L_p || R_p \]

\[ Q = \frac{R_p}{w_0L_p} \]

- Equate real and imaginary parts of the left and right expressions (so that \( Z_{in} \) is the same for both)
  - Also equate Q values

\[ R_p = R_s(Q^2 + 1) \approx R_sQ^2 \quad (for \quad Q \gg 1) \]

\[ L_p = L_s\left(\frac{Q^2 + 1}{Q^2}\right) \approx L_s \quad (for \quad Q \gg 1) \]
Comparison of Series and Parallel RC Circuits

- Equate real and imaginary parts of the left and right expressions (so that $Z_{in}$ is the same for both)
  - Also equate Q values

$$R_p = R_s(Q^2 + 1) \approx R_s Q^2 \quad (\text{for } Q \gg 1)$$

$$C_p = C_s \left( \frac{Q^2}{Q^2 + 1} \right) \approx C_s \quad (\text{for } Q \gg 1)$$
Example Transformation to Parallel RLC

- Assume $Q >> 1$

Series to Parallel Transformation

- Note at resonance:

\[
Z_{in} = R_p \approx Q^2 R_s \quad \text{(purely real)}
\]

\[
L_p \approx L_s
\]

\[
R_p \approx Q^2 R_s
\]
Tapped Capacitor as a Transformer

To first order:

\[
\frac{R_{in}}{R_L} \approx \left( \frac{C_1 + C_2}{C_1} \right)^2
\]

- We will see this used in Colpitts oscillator
This type of oscillator structure is quite popular in current CMOS implementations

- Advantages
  - Simple topology
  - Differential implementation (good for feeding differential circuits)
  - Good phase noise performance can be achieved
Derive a parallel RLC network that includes the loss of the tank inductor and capacitor
- Typically, such loss is dominated by series resistance in the inductor
Analysis of Negative Resistance Oscillator (Step 2)

- Split oscillator circuit into half circuits to simplify analysis
  - Leverages the fact that we can approximate $V_s$ as being incremental ground (this is not quite true, but close enough)
- Recognize that we have a diode connected device with a negative transconductance value
  - Replace with negative resistor
    - Note: $G_m$ is *large signal* transconductance value
Design of Negative Resistance Oscillator

- Design tank components to achieve high Q
  - Resulting $R_p$ value is as large as possible
- Choose bias current ($I_{bias}$) for large swing (without going far into saturation)
  - We’ll estimate swing as a function of $I_{bias}$ shortly
- Choose transistor size to achieve adequately large $g_{m1}$
  - Usually twice as large as $1/R_{p1}$ to guarantee startup
Calculation of Oscillator Swing

- Design tank components to achieve high Q
  - Resulting $R_p$ value is as large as possible
- Choose bias current ($I_{bias}$) for large swing (without going far into saturation)
  - We’ll estimate swing as a function of $I_{bias}$ in next slide
- Choose transistor size to achieve adequately large $g_{m1}$
  - Usually twice as large as $1/R_{p1}$ to guarantee startup
Calculation of Oscillator Swing as a Function of $I_{bias}$

- By symmetry, assume $I_1(t)$ is a square wave
  - We are interested in determining fundamental component
    - (DC and harmonics filtered by tank)

- Fundamental component is
  $$ I_1(t) \bigg|_{\text{fundamental}} = \frac{2}{\pi} I_{bias} \sin(w_0 t), \quad \text{where} \quad w_0 = \frac{2\pi}{T} $$

- Resulting oscillator amplitude
  $$ A = \frac{2}{\pi} I_{bias} R_p $$
Variations on a Theme

- **Biasing can come from top or bottom**
- **Can use either NMOS, PMOS, or both for transconductor**
  - Use of both NMOS and PMOS for coupled pair would appear to achieve better phase noise at a given power dissipation
Colpitts Oscillator

- Carryover from discrete designs in which single-ended approaches were preferred for simplicity
  - Achieves negative resistance with only one transistor
  - Differential structure can also be implemented
- Good phase noise can be achieved, but not apparent there is an advantage of this design over negative resistance design for CMOS applications
Analysis of Cap Transformer used in Colpitts

Voltage drop across $R_L$ is reduced by capacitive voltage divider
- Assume that impedances of caps are less than $R_L$ at resonant frequency of tank (simplifies analysis)
  - Ratio of $V_1$ to $V_{out}$ set by caps and not $R_L$
- Power conservation leads to transformer relationship shown
## Simplified Model of Colpitts

- **Purpose of cap transformer**
  - Reduces loading on tank
  - Reduces swing at source node (important for bipolar version)

- **Transformer ratio set to achieve best noise performance**
Design of Colpitts Oscillator

- Design tank for high Q
- Choose bias current \( I_{\text{bias}} \) for large swing (without going far into saturation)
- Choose transformer ratio for best noise
  - Rule of thumb: choose \( N = 1/5 \) according to Tom Lee
- Choose transistor size to achieve adequately large \( g_{m1} \)
Calculation of Oscillator Swing as a Function of $I_{\text{bias}}$

- $I_1(t)$ consists of pulses whose shape and width are a function of the transistor behavior and transformer ratio
  - Approximate as narrow square wave pulses with width $W$

- Fundamental component is
  \[
  I_1(t) \bigg|_{\text{fundamental}} \approx 2I_{\text{bias}} \sin(w_0 t), \quad \text{where} \quad w_0 = \frac{2\pi}{T}
  \]

- Resulting oscillator amplitude
  \[
  A \approx 2I_{\text{bias}}R_{eq}
  \]
Clapp Oscillator

- Same as Colpitts except that inductor portion of tank is isolated from the drain of the device
  - Allows inductor voltage to achieve a larger amplitude without exceeded the max allowable voltage at the drain
    - Good for achieving lower phase noise
Hartley Oscillator

- Same as Colpitts, but uses a tapped inductor rather than series capacitors to implement the transformer portion of the circuit
  - Not popular for IC implementations due to the fact that capacitors are easier to realize than inductors
Integrated Resonator Structures

- Inductor and capacitor tank
  - Lateral caps have high Q (> 50)
  - Spiral inductors have moderate Q (5 to 10), but completely integrated and have tight tolerance (< § 10%)
  - Bondwire inductors have high Q (> 40), but not as “integrated” and have poor tolerance (> § 20%)
Integrated Resonator Structures

- **Integrated transformer**
  - Leverages self and mutual inductance for resonance to achieve higher Q
Quarter Wave Resonator

- Impedance calculation (from Lecture 4)

\[ Z\left(\frac{\lambda_0}{4}\right) \approx -j \frac{2}{\pi} \sqrt{\frac{L}{C}} \left( \frac{\omega_0}{\Delta \omega} \right) \]

- Looks like parallel LC tank!

- Benefit – very high Q can be achieved with fancy dielectric

- Negative – relatively large area (external implementation in the past), but getting smaller with higher frequencies!
Other Types of Resonators

- Quartz crystal
  - Very high Q, and very accurate and stable resonant frequency
    - Confined to low frequencies (< 200 MHz)
    - Non-integrated
  - Used to create low noise, accurate, “reference” oscillators
- SAW devices
  - High frequency, but poor accuracy (for resonant frequency)
- MEMS devices
  - Cantilever beams – promise high Q, but non-tunable and haven’t made it to the GHz range, yet, for resonant frequency
  - FBAR – Q > 1000, but non-tunable and poor accuracy
  - More on this topic in the last lecture this week
Voltage Controlled Oscillators (VCO’s)

- Include a tuning element to adjust oscillation frequency
  - Typically use a variable capacitor (varactor)
- Varactor incorporated by replacing fixed capacitance
  - Note that much fixed capacitance cannot be removed (transistor junctions, interconnect, etc.)
  - Fixed cap lowers frequency tuning range
Model VCO in a small signal manner by looking at deviations in frequency about the bias point

- Assume linear relationship between input voltage and output frequency

\[ F_{out}(t) = K_v v_{in}(t) \]
**Model for Voltage to Phase Mapping of VCO**

\[
F_{out}(t) = K_v v_{in}(t)
\]

- Phase is more convenient than frequency for analysis
  - The two are related through an integral relationship

\[
\Phi_{out}(t) = \int_{-\infty}^{t} 2\pi F_{out}(\tau) d\tau = \int_{-\infty}^{t} 2\pi K_v v_{in}(\tau) d\tau
\]

- Intuition of integral relationship between frequency and phase
**Frequency Domain Model of VCO**

- Take Laplace Transform of phase relationship

\[
\Phi_{out}(t) = \int_{-\infty}^{t} 2\pi K_v v_{in}(\tau) d\tau
\]

\[\Rightarrow \quad \Phi_{out}(s) = 2\pi K_v v_{in}(s)\]

- Note that $K_v$ is in units of $\text{Hz/V}$
Varactor Implementation – Diode Version

- Consists of a reverse biased diode junction
  - Variable capacitor formed by depletion capacitance
  - Capacitance drops as roughly the square root of the bias voltage
- Advantage – can be fully integrated in CMOS
- Disadvantages – low Q (often < 20), and low tuning range (§ 20%)
A Recently Popular Approach – The MOS Varactor

- Consists of a MOS transistor (NMOS or PMOS) with drain and source connected together
  - Abrupt shift in capacitance as inversion channel forms
- Advantage – easily integrated in CMOS
- Disadvantage – Q is relatively low in the transition region
  - Note that large signal is applied to varactor – transition region will be swept across each VCO cycle
A Method To Increase Q of MOS Varactor

- High Q metal caps are switched in to provide coarse tuning
- Low Q MOS varactor used to obtain fine tuning
Supply Pulling and Pushing

- Supply voltage has an impact on the VCO frequency
  - Voltage across varactor will vary, thereby causing a shift in its capacitance
  - Voltage across transistor drain junctions will vary, thereby causing a shift in its depletion capacitance
- This problem is addressed by building a supply regulator specifically for the VCO
Noise close in frequency to VCO resonant frequency can cause VCO frequency to shift when its amplitude becomes high enough.
Example of Injection Locking

- For homodyne systems, VCO frequency can be very close to that of interferers

- Injection locking can happen if inadequate isolation from mixer RF input to LO port

- Follow VCO with a buffer stage with high reverse isolation to alleviate this problem
Summary

- Several concepts are useful for understanding LC oscillators
  - Barkhausen criterion
  - Impedance transformations

- Voltage-controlled oscillators incorporate a tunable element such as varactor
  - Increased range achieved by using switched capacitor network for coarse tuning
    - Improves varactor Q, as well

- Several things to watch out for
  - Supply pulling, injection locking, coupling
Noise in Voltage Controlled Oscillators
**VCO Noise in Wireless Systems**

- **VCO noise has a negative impact on system performance**
  - Receiver – lower sensitivity, poorer blocking performance
  - Transmitter – increased spectral emissions (output spectrum must meet a mask requirement)
- **Noise is characterized in frequency domain**
VCO noise also has a negative impact on data links
- Receiver – increases bit error rate (BER)
- Transmitter – increases jitter on data stream (transmitter must have jitter below a specified level)

Noise is characterized in the time domain
Outline of Talk

- System level view of VCO and PLL noise
- Linearized model of VCO noise
  - Noise figure
  - Equipartition theorem
  - Leeson’s formula
- Cyclo-stationary view of VCO noise
  - Hajimiri model
- Back to Leeson’s formula
Noise Sources Impacting VCO

- **Extrinsic noise**
  - Noise from other circuits (including PLL)
- **Intrinsic noise**
  - Noise due to the VCO circuitry
VCO Model for Noise Analysis

We will focus on phase noise (and its associated jitter)
- Model as phase signal in output sine waveform

\[ \text{out}(t) = 2 \cos(2\pi f_0 t + \Phi_{\text{out}}(t)) \]
**Simplified Relationship Between \( \Phi_{out} \) and Output**

Using a familiar trigonometric identity:

\[
\text{out}(t) = 2 \cos(2\pi f_0 t + \Phi_{out}(t))
\]

**Given that the phase noise is small**

\[
\cos(\Phi_{out}(t)) \approx 1, \quad \sin(\Phi_{out}(t)) \approx \Phi_{out}(t)
\]

\[
\Rightarrow \quad \text{out}(t) = 2 \cos(2\pi f_0 t) - 2 \sin(2\pi f_0 t) \Phi_{out}(t)
\]
Calculation of Output Spectral Density

\[ \text{out}(t) = 2 \cos(2\pi f_o t) - 2 \sin(2\pi f_o t) \Phi_{\text{out}}(t) \]

- Calculate autocorrelation

\[ R\{ \text{out}(t) \} = R\{ 2 \cos(2\pi f_o t) \} + R\{ 2 \sin(2\pi f_o t) \} \cdot R\{ \Phi_{\text{out}}(t) \} \]

- Take Fourier transform to get spectrum

\[ S_{\text{out}}(f) = S_{\text{sin}}(f) + S_{\text{sin}}(f) \ast S_{\Phi_{\text{out}}} \]

- Note that * symbol corresponds to convolution

- In general, phase spectral density can be placed into one of two categories
  - Phase noise – \( \Phi_{\text{out}}(t) \) is non-periodic
  - Spurious noise - \( \Phi_{\text{out}}(t) \) is periodic
Output Spectrum with Phase Noise

- Suppose input noise to VCO ($v_n(t)$) is bandlimited, non-periodic noise with spectrum $S_{vn}(f)$
  - In practice, derive phase spectrum as
    \[
    S_{\Phi_{out}}(f) = \left( \frac{K_v}{f} \right)^2 S_{vn}(f)
    \]
- Resulting output spectrum

\[
S_{out}(f) = S_{sin}(f) + S_{sin}(f) \ast S_{\Phi_{out}}
\]
Measurement of Phase Noise in dBc/Hz

- Definition of $L(f)$

\[
L(f) = 10 \log \left( \frac{\text{Spectral density of noise}}{\text{Power of carrier}} \right)
\]

- Units are dBc/Hz

- For this case

\[
L(f) = 10 \log \left( \frac{2S_{\Phi_{out}}(f)}{2} \right) = 10 \log(S_{\Phi_{out}}(f))
\]

- Valid when $\Phi_{out}(t)$ is small in deviation (i.e., when carrier is not modulated, as currently assumed)
**Single-Sided Version**

- Definition of \( L(f) \) remains the same

\[
L(f) = 10 \log \left( \frac{\text{Spectral density of noise}}{\text{Power of carrier}} \right)
\]

- Units are dBc/Hz

- For this case

\[
L(f) = 10 \log \left( \frac{S_{\Phi out}(f)}{1} \right) = 10 \log(S_{\Phi out}(f))
\]

- So, we can work with either one-sided or two-sided spectral densities since \( L(f) \) is set by ratio of noise density to carrier power
Output Spectrum with Spurious Noise

- Suppose input noise to VCO is

\[ v_n(t) = \frac{d_{spur}}{K_v} \cos(2\pi f_{spur}t) \]

\[ \Rightarrow \Phi_{out}(t) = 2\pi K_v \int v_n(t) \, dt = \frac{d_{spur}}{f_{spur}} \sin(2\pi f_{spur}t) \]

- Resulting output spectrum

\[ S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}} \]
Measurement of Spurious Noise in dBc

- Definition of dBc

\[ 10 \log \left( \frac{\text{Power of tone}}{\text{Power of carrier}} \right) \]

- We are assuming double sided spectra, so integrate over positive and negative frequencies to get power
  - Either single or double-sided spectra can be used in practice

- For this case

\[ 10 \log \left( \frac{2 \left( \frac{d_{spur}}{2f_{spur}} \right)^2}{2} \right) = 20 \log \left( \frac{d_{spur}}{2f_{spur}} \right) dBc \]
Noise sources in oscillators are put in two categories
- Noise due to tank loss
- Noise due to active negative resistance

We want to determine how these noise sources influence the phase noise of the oscillator
Equivalent Model for Noise Calculations

Active Negative Resistance

Noise Due to Active Negative Resistance

Compensated Resonator with Noise from Tank

Noise Due to Active Negative Resistance

Noise from Tank

Ideal Tank
Calculate Impedance Across Ideal LC Tank Circuit

\[ Z_{\text{tank}}(w) = \frac{1}{j \omega C_p} || j \omega L_p = \frac{j \omega L_p}{1 - \omega^2 L_p C_p} \]

- Calculate input impedance about resonance

Consider \( w = \omega_o + \Delta w \), where \( \omega_o = \frac{1}{\sqrt{L_p C_p}} \)

\[ Z_{\text{tank}}(\Delta w) = \frac{j (\omega_o + \Delta w) L_p}{1 - (\omega_o + \Delta w)^2 L_p C_p} \]

\[ = \frac{j (\omega_o + \Delta w) L_p}{1 - \omega_o^2 L_p C_p - 2 \Delta w (\omega_o L_p C_p) - \Delta w^2 L_p C_p} \approx \frac{j (\omega_o + \Delta w) L_p}{-2 \Delta w (\omega_o L_p C_p)} \]

\[ \Rightarrow Z_{\text{tank}}(\Delta w) \approx \frac{j \omega_o L_p}{-2 \Delta w (\omega_o L_p C_p)} = -\frac{j}{2 \omega_o C_p} \left( \frac{\omega_o}{\Delta w} \right) \]

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A Convenient Parameterization of LC Tank Impedance

- Actual tank has loss that is modeled with $R_p$
  - Define $Q$ according to actual tank
    \[
    Q = R_p w_o C_p \quad \Rightarrow \quad \frac{1}{w_o C_p} = \frac{R_p}{Q}
    \]
- Parameterize ideal tank impedance in terms of $Q$ of actual tank
  \[
  Z_{tank}(\Delta w) \approx -\frac{j}{2 w_o C_p} \left( \frac{w_o}{\Delta w} \right)
  \]
  \[
  \Rightarrow \quad |Z_{tank}(\Delta f)|^2 \approx \left( \frac{R_p f_o}{2Q \Delta f} \right)^2
  \]
Overall Noise Output Spectral Density

Assume noise from active negative resistance element and tank are uncorrelated

\[
\frac{v_{out}^2}{\Delta f} = \left( \frac{i_{nRp}^2}{\Delta f} + \frac{i_{nRn}^2}{\Delta f} \right) |Z_{tank}(\Delta f)|^2
\]

\[
= \frac{i_{nRp}^2}{\Delta f} \left( 1 + \frac{i_{nRn}^2}{\Delta f} \right) \left/ \frac{i_{nRp}^2}{\Delta f} \right) |Z_{tank}(\Delta f)|^2
\]

Note that the above expression represents total noise that impacts both amplitude and phase of oscillator output
Parameterize Noise Output Spectral Density

- Noise Due to Active Negative Resistance
- Noise from Tank

From previous slide

\[
\frac{v_{out}^2}{\Delta f} = \frac{i_{nRp}^2}{\Delta f} \left( 1 + \frac{i_{nRn}^2}{\Delta f} / \frac{i_{nRp}^2}{\Delta f} \right) |Z_{tank}(\Delta f)|^2
\]

\[F(\Delta f)\]

- \(F(\Delta f)\) is defined as

\[
F(\Delta f) = \frac{\text{total noise in tank at frequency } \Delta f}{\text{noise in tank due to tank loss at frequency } \Delta f}
\]
Noise from tank is due to resistor $R_p$

$$\frac{i_{nR_p}^2}{\Delta f} = 4kT \frac{1}{R_p} \quad \text{(single-sided spectrum)}$$

$Z_{\text{tank}}(\Delta f)$ found previously

$$|Z_{\text{tank}}(\Delta f)|^2 \approx \left( \frac{R_p f_o}{2Q \Delta f} \right)^2$$

Output noise spectral density expression (single-sided)

$$\frac{v_{out}^2}{\Delta f} = 4kT \frac{1}{R_p} F(\Delta f) \left( \frac{R_p f_o}{2Q \Delta f} \right)^2 = 4kT F(\Delta f) R_p \left( \frac{1}{2Q \Delta f} \right)^2$$
Separation into Amplitude and Phase Noise

- **Equipartition theorem** states that noise impact splits evenly between amplitude and phase for $V_{\text{sig}}$ being a sine wave.
  - Amplitude variations suppressed by feedback in oscillator

\[
\frac{\overline{v_{\text{out}}^2}}{\Delta f} \bigg|_{\text{phase}} = 2kT F(\Delta f) R_p \left( \frac{1}{2Q \Delta f} \right)^2 \quad \text{(single-sided)}
\]
**Output Phase Noise Spectrum (Leeson’s Formula)**

\[ L(\Delta f) = 10 \log \left( \frac{\text{Spectral density of noise}}{\text{Power of carrier}} \right) \]

- All power calculations are referenced to the tank loss resistance, \( R_p \)

\[ P_{sig} = \frac{V_{sig,rms}^2}{R_p} = \frac{(A/\sqrt{2})^2}{R_p}, \quad S_{noise}(\Delta f) = \frac{1}{R_p \Delta f} \frac{v_{out}^2}{\Delta f} \]

\[ L(\Delta f) = 10 \log \left( \frac{S_{noise}(\Delta f)}{P_{sig}} \right) = 10 \log \left( \frac{2kTF(\Delta f)}{P_{sig}} \left( \frac{1}{2Q \Delta f} \right)^2 \right) \]
Example: Active Noise Same as Tank Noise

- **Noise factor for oscillator in this case is**

  \[
  F(\Delta f) = 1 + \frac{i_{nR_n}^2}{\Delta f} / \frac{i_{nR_p}^2}{\Delta f} = 2
  \]

- **Resulting phase noise**

  \[
  L(\Delta f) = 10 \log \left( \frac{4kT}{P_{sig}} \left( \frac{1}{2Q \Delta f} \right)^2 \right)
  \]
**The Actual Situation is Much More Complicated**

- Impact of tank generated noise easy to assess
- Impact of transistor generated noise is complicated
  - Noise from $M_1$ and $M_2$ is modulated on and off
  - Noise from $M_3$ is modulated before influencing $V_{out}$
  - Transistors have 1/f noise
- Also, transistors can degrade Q of tank
Phase noise drops at -20 dB/decade over a wide frequency range, but deviates from this at:
- Low frequencies – slope increases (often -30 dB/decade)
- High frequencies – slope flattens out (oscillator tank does not filter all noise sources)

Frequency breakpoints and magnitude scaling are not readily predicted by the analysis approach taken so far.
Leeson proposed an ad hoc modification of the phase noise expression to capture the above noise profile.

\[
L(\Delta f) = 10 \log \left( \frac{2FkT}{P_{\text{sig}}} \left( 1 + \left( \frac{1}{2Q \Delta f} \right)^2 \right) \left( 1 + \frac{\Delta f_1/f^3}{|\Delta f|} \right) \right)
\]

Note: he assumed that \( F(\Delta f) \) was constant over frequency.
Our concern is what happens when noise current produces a voltage across the tank
- Such voltage deviations give rise to both amplitude and phase noise
- Amplitude noise is suppressed through feedback (or by amplitude limiting in following buffer stages)
  - Our main concern is phase noise

We argued that impact of noise divides equally between amplitude and phase for sine wave outputs
- What happens when we have a non-sine wave output?
Characterize impact of current noise on amplitude and phase through their associated impulse responses

- Phase deviations are accumulated
- Amplitude deviations are suppressed
If we vary the time at which the current impulse is injected, its impact on phase and amplitude changes.

- Need a time-varying model
Illustration of Time-Varying Impact of Noise on Phase

- High impact on phase when impulse occurs close to the zero crossing of the VCO output
- Low impact on phase when impulse occurs at peak of output
Define Impulse Sensitivity Function (ISF) – $\Gamma(2\pi f_o t)$

- ISF constructed by calculating phase deviations as impulse position is varied
  - Observe that it is periodic with same period as VCO output
Parameterize Phase Impulse Response in Terms of ISF

\[ h_{\Phi}(t, \tau) = \frac{\Gamma(2\pi f_o \tau)}{q_{max}} u(t - \tau) \]
Examples of ISF for Different VCO Output Waveforms

- **ISF (i.e., \( \Gamma \)) is approximately proportional to derivative of VCO output waveform**
  - Its magnitude indicates where VCO waveform is most sensitive to noise current into tank with respect to creating phase noise
- **ISF is periodic**
- **In practice, derive it from simulation of the VCO**
Phase Noise Analysis Using LTV Framework

- Computation of phase deviation for an arbitrary noise current input

\[
\Phi_{out}(t) = \int_{-\infty}^{\infty} h_{\Phi}(t, \tau) i_n(\tau) d\tau = \frac{1}{q_{max}} \int_{-\infty}^{t} \Gamma(2\pi f_o \tau) i_n(\tau) d\tau
\]

- Analysis simplified if we describe ISF in terms of its Fourier series (note: \( c_o \) here is different than book)

\[
\Gamma(2\pi f_o \tau) = \frac{c_o}{\sqrt{2}} + \sum_{n=1}^{\infty} c_n \cos(n2\pi f_o \tau + \theta_n)
\]

\[
\Rightarrow \Phi_{out}(t) = \int_{-\infty}^{t} \left( \frac{c_o}{\sqrt{2}} + \sum_{n=1}^{\infty} c_n \cos(n2\pi f_o \tau + \theta_n) \right) \frac{i_n(\tau)}{q_{max}} d\tau
\]
- Noise from current source is mixed down from different frequency bands and scaled according to ISF coefficients.
Phase Noise Calculation for White Noise Input (Part 1)

Note that \( \frac{i_n^2}{\Delta f} \) is the single-sided noise spectral density of \( i_n(t) \).

\[
S_X(f) = \left( \frac{1}{q_{\text{max}}} \right)^2 \frac{i_n^2}{2\Delta f}
\]

\( f = f_0 \)

- \( S_A(f) \)
- \( S_B(f) \)
- \( S_C(f) \)
- \( S_D(f) \)
Phase Noise Calculation for White Noise Input (Part 2)

\[ S_{\Phi_{\text{out}}} (f) = \left| \frac{1}{j2\pi f} \right|^2 \left( \left( \frac{c_0}{2} \right)^2 S_A(f) + \left( \frac{c_1}{2} \right)^2 S_B(f) + \cdots \right) \]
Spectral Density of Phase Signal

- From the previous slide

\[ S_{\Phi_{out}}(f) = \left( \frac{1}{2\pi f} \right)^2 \left( \left( \frac{c_o}{2} \right)^2 S_A(f) + \left( \frac{c_1}{2} \right)^2 S_B(f) + \cdots \right) \]

- Substitute in for \( S_A(f) \), \( S_B(f) \), etc.

\[ S_{\Phi_{out}}(f) = \left( \frac{1}{2\pi f} \right)^2 \left( \left( \frac{c_o}{2} \right)^2 + \left( \frac{c_1}{2} \right)^2 + \cdots \right) 2 \left( \frac{1}{q_{max}} \right)^2 \frac{\bar{i}_n^2}{2\Delta f} \]

- Resulting expression

\[ S_{\Phi_{out}}(f) = \left( \frac{1}{2\pi f} \right)^2 \left( \sum_{n=0}^{\infty} c_n^2 \right) \frac{1}{4} \left( \frac{1}{q_{max}} \right)^2 \frac{\bar{i}_n^2}{\Delta f} \]
Output Phase Noise

We now know

\[
S_{\Phi_{out}}(f) = \left| \frac{1}{2\pi f} \right|^2 \left( \sum_{n=0}^{\infty} c_n^2 \right) \frac{1}{4} \left( \frac{1}{q_{max}} \right)^2 \frac{i_n^2}{\Delta f}
\]

\[
L(\Delta f) = 10 \log(S_{\Phi_{out}}(\Delta f))
\]

Resulting phase noise

\[
L(\Delta f) = 10 \log \left( \left( \frac{1}{2\pi \Delta f} \right)^2 \left( \sum_{n=0}^{\infty} c_n^2 \right) \frac{1}{4} \left( \frac{1}{q_{max}} \right)^2 \frac{i_n^2}{\Delta f} \right)
\]
The Impact of $1/f$ Noise in Input Current (Part 1)

Note that $\frac{i_n^2}{\Delta f}$ is the single-sided noise spectral density of $i_n(t)$.

$$S_X(f) = \frac{1}{q_{\text{max}}} \frac{i_n^2}{2\Delta f}$$
The Impact of 1/f Noise in Input Current (Part 2)

\[ S_{\Phi_{out}}(f) \bigg|_{1/f^3} = \left| \frac{1}{j2\pi f} \right|^2 \left( \frac{c_0}{2} \right)^2 S_A(f) \]
Calculation of Output Phase Noise in 1/f³ region

- From the previous slide

\[ S_{\Phi_{out}}(f) \bigg|_{1/f^3} = \left( \frac{1}{2\pi f} \right)^2 \left( \frac{c_o}{2} \right)^2 S_A(f) \]

- Assume that input current has 1/f noise with corner frequency \( f_{1/f} \)

\[ S_A(f) = \left( \frac{1}{q_{max}} \right)^2 \frac{i_n^2}{\Delta f} \left( \frac{f_{1/f}}{\Delta f} \right) \]

- Corresponding output phase noise

\[ L(\Delta f) \bigg|_{1/f^3} = 10 \log \left( \left( \frac{1}{2\pi \Delta f} \right)^2 \left( \frac{c_o}{2} \right)^2 S_A(f) \right) \]

\[ = 10 \log \left( \left( \frac{1}{2\pi \Delta f} \right)^2 \left( \frac{c_o^2}{4} \right) \frac{1}{4} \left( \frac{1}{q_{max}} \right)^2 \frac{i_n^2}{\Delta f} \left( \frac{f_{1/f}}{\Delta f} \right) \right) \]
Calculation of $1/f^3$ Corner Frequency

\[ L(\Delta f) = 10 \log \left( \frac{1}{2\pi \Delta f} \right)^2 \left( c_0^2 \right) \frac{1}{4} \left( \frac{1}{q_{max}} \right)^2 \frac{i_n^2}{\Delta f} \left( \frac{f_{1/f}}{\Delta f} \right) \]

\[ L(\Delta f) = 10 \log \left( \frac{1}{2\pi \Delta f} \right)^2 \left( \sum_{n=0}^{\infty} c_n^2 \right) \frac{1}{4} \left( \frac{1}{q_{max}} \right)^2 \frac{i_n^2}{\Delta f} \]

\( (A) = (B) \) at:

\[ \Delta f_{1/f^3} = \left( c_0^2 / \sum_{n=0}^{\infty} c_n^2 \right) f_{1/f} \]
Impact of Oscillator Waveform on $1/f^3$ Phase Noise

- Key Fourier series coefficient of ISF for $1/f^3$ noise is $c_o$
  - If DC value of ISF is zero, $c_o$ is also zero
- For symmetric oscillator output waveform
  - DC value of ISF is zero – no upconversion of flicker noise!
    (i.e. output phase noise does not have $1/f^3$ region)
- For asymmetric oscillator output waveform
  - DC value of ISF is nonzero – flicker noise has impact
**Issue – We Have Ignored Modulation of Current Noise**

- In practice, transistor generated noise is modulated by the varying bias conditions of its associated transistor.
  - As transistor goes from saturation to triode to cutoff, its associated noise changes dramatically.
- Can we include this issue in the LTV framework?
**Inclusion of Current Noise Modulation**

- **Recall**

\[
\Phi_{out}(t) = \int_{-\infty}^{\infty} h_{\Phi}(t, \tau) i_n(\tau) d\tau = \frac{1}{q_{max}} \int_{-\infty}^{t} \Gamma(2\pi f_o \tau) i_{in}(\tau) d\tau
\]

- **By inspection of figure**

\[
\Rightarrow \quad \Phi_{out}(t) = \frac{1}{q_{max}} \int_{-\infty}^{t} \Gamma(2\pi f_o \tau) \alpha(2\pi f_o \tau) i_{in}(\tau) d\tau
\]

- **We therefore apply previous framework with ISF as**

\[
\Gamma_{eff}(2\pi f_o \tau) = \Gamma(2\pi f_o \tau) \alpha(2\pi f_o \tau)
\]
**Placement of Current Modulation for Best Phase Noise**

- **Phase noise expression (ignoring 1/f noise)**

\[
L(\Delta f) = 10 \log \left( \left( \frac{1}{2\pi \Delta f} \right)^2 \left( \sum_{n=0}^{\infty} c_n^2 \right) \frac{1}{4} \left( \frac{1}{q_{max}} \right)^2 \frac{\dot{v}_n^2}{\Delta f} \right)
\]

- **Minimum phase noise achieved by minimizing sum of square of Fourier series coefficients (i.e. rms value of \(\Gamma_{eff}\))**
Colpitts Oscillator Provides Optimal Placement of $\alpha$

- Current is injected into tank at bottom portion of VCO swing
  - Current noise accompanying current has minimal impact on VCO output phase
Step 1: calculate the impulse sensitivity function of each oscillator noise source using a simulator

Step 2: calculate the noise current modulation waveform for each oscillator noise source using a simulator

Step 3: combine above results to obtain $\Gamma_{\text{eff}}(2\pi f_o t)$ for each oscillator noise source

Step 4: calculate Fourier series coefficients for each $\Gamma_{\text{eff}}(2\pi f_o t)$

Step 5: calculate spectral density of each oscillator noise source (before modulation)

Step 6: calculate overall output phase noise using the results from Step 4 and 5 and the phase noise expressions derived in this lecture (or the book)
Alternate Approach for Negative Resistance Oscillator

- Recall Leeson’s formula

\[ L(\Delta f) = 10 \log \left( \frac{2kTF(\Delta f)}{P_{sig}} \left( \frac{1}{2Q} \frac{f_o}{\Delta f} \right)^2 \right) \]

- Key question: how do you determine \( F(\Delta f) \)?
F(Δf) Has Been Determined for This Topology

- Rael et. al. have come up with a closed form expression for F(Δf) for the above topology
- In the region where phase noise falls at -20 dB/dec:

\[
F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A} + \gamma \frac{4}{9} g_{do,M3} R_p \quad (R_p = R_{p1} = R_{p2})
\]

M.H. Perrott
References to Rael Work

- **Phase noise analysis**

- **Implementation**
Designing for Minimum Phase Noise

To achieve minimum phase noise, we’d like to minimize $F(\Delta f)$.

The above formulation provides insight of how to do this:
- Key observation: (C) is often quite significant.

(A) Noise from tank resistance
(B) Noise from $M_1$ and $M_2$
(C) Noise from $M_3$
Elimination of Component (C) in F(Δf)

- Capacitor $C_f$ shunts noise from $M_3$ away from tank
  - Component (C) is eliminated!
- Issue – impedance at node $V_s$ is very low
  - Causes $M_1$ and $M_2$ to present a low impedance to tank during portions of the VCO cycle
    - $Q$ of tank is degraded
Use Inductor to Increase Impedance at Node $V_s$

- Voltage at node $V_s$ is a rectified version of oscillator output
  - Fundamental component is at twice the oscillation frequency
- Place inductor between $V_s$ and current source
  - Choose value to resonate with $C_f$ and parasitic source capacitance at frequency $2f_o$
- Impedance of tank not degraded by $M_1$ and $M_2$
  - $Q$ preserved!
Let’s now focus on component (B)
- Depends on bias current and oscillation amplitude

\[
F(\Delta f) = 1 + \frac{2\gamma I_{bias}R_P}{\pi A} + \frac{4}{9\gamma g_{ds, M3}R_P}
\]

(A) Noise from tank resistance
(B) Noise from M₁ and M₂
(C) Noise from M₃
Minimization of Component (B) in $F(\Delta f)$

$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A}$

(B)

- Recall from Lecture 11

  $A = \frac{2}{\pi} I_{bias} R_p$

  $\Rightarrow F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi (2/\pi) I_{bias} R_p} = 1 + \gamma$

- So, it would seem that $I_{bias}$ has no effect!
  - Not true – want to maximize $A$ (i.e. $P_{sig}$) to get best phase noise, as seen by:

  $$L(\Delta f) = 10 \log \left( \frac{2kT F(\Delta f)}{P_{sig}} \left( \frac{1}{2Q \Delta f} \right)^2 \right)$$
Current-Limited Versus Voltage-Limited Regimes

- Oscillation amplitude, $A$, cannot be increased above supply imposed limits
- If $I_{bias}$ is increased above the point that $A$ saturates, then (B) increases

- Current-limited regime: amplitude given by
- Voltage-limited regime: amplitude saturated

Best phase noise achieved at boundary between these regimes!

$F(\Delta f) = 1 + \frac{2 \gamma I_{bias} R_p}{\pi A}$

(B)
Summary

- Leeson’s model is outcome of linearized VCO noise analysis
- Hajimiri method provides insights into cyclostationary behavior, 1/f noise upconversion and impact of noise current modulation
- Rael method useful for CMOS negative-resistance topology
  - Closed form solution of phase noise!
  - Provides a great deal of design insight
- Practical VCO phase noise analysis is done through simulation these days
  - Spectre RF from Cadence, FastSpice from Berkeley Design Automation is often utilized to estimate phase noise for integrated oscillators