Notation for Mean, Variance, and Correlation

- Consider random variables $x$ and $y$ with probability density functions $f_x(x)$ and $f_y(y)$ and joint probability function $f_{xy}(x,y)$
  - Expected value (mean) of $x$ is
    \[ \overline{x} = E(x) = \int_{-\infty}^{\infty} x f_x(x) \, dx \]
    - Note: we will often abuse notation and denote $\overline{x}$ as a random variable (i.e., noise) rather than its mean
  - The variance of $x$ (assuming it has zero mean) is
    \[ \overline{x^2} = E(x^2) = \int_{-\infty}^{\infty} x^2 f_x(x) \, dx \]
  - A useful statistic is
    \[ \overline{xy} = E(xy) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f_{xy}(x,y) \, dx \, dy \]
    - If the above is zero, $x$ and $y$ are said to be uncorrelated
**Relationship Between Variance and Spectral Density**

- **Two-sided spectrum**
  \[ \overline{\sigma^2} = \int_{-f_2}^{-f_1} S_x(f) df + \int_{f_1}^{f_2} S_x(f) df \]
  - Since spectrum is symmetric
  \[ \Rightarrow \overline{\sigma^2} = 2 \int_{f_1}^{f_2} S_x(f) df \]

- **One-sided spectrum defined over positive frequencies**
  - Magnitude defined as twice that of its corresponding two-sided spectrum
- In the next few lectures, we assume a one-sided spectrum for all noise analysis
For the random signal passing through a linear, time-invariant system with transfer function $H(f)$

$$S_y(f) = |H(f)|^2 S_x(f)$$

- We see that if $x(t)$ is amplified by gain $A$, we have

$$S_y(f) = A^2 S_x(f) \Rightarrow \overline{y^2} = A^2 \overline{x^2}$$
Noise in Resistors

- Can be described in terms of either voltage or current

\[ \overline{e_n^2} = 4kTR\Delta f \]
\[ \overline{i_n^2} = 4kT \frac{1}{R} \Delta f \]

- \( k \) is Boltzmann’s constant

\[ k = 1.38 \times 10^{-23} \text{ J/K} \]

- \( T \) is temperature (in Kelvins)
  - Usually assume room temperature of 27 degrees Celsius

\[ \Rightarrow T = 300K \]
Noise In Inductors and Capacitors

- Ideal capacitors and inductors have no noise!
- In practice, however, they will have parasitic resistance
  - Induces noise
  - Parameterized by adding resistances in parallel/series with inductor/capacitor
    - Include parasitic resistor noise sources
Noise in CMOS Transistors (Assumed in Saturation)

Modeling of noise in transistors must include several noise sources

- Drain noise
  - Thermal and 1/f – influenced by transistor size and bias

- Gate noise
  - Induced from channel – influenced by transistor size and bias
  - Caused by routing resistance to gate (including resistance of polysilicon gate)
    - Can be made negligible with proper layout such as fingering of devices

Transistor Noise Sources

- Drain Noise (Thermal and 1/f)
- Gate Noise (Induced and Routing Parasitic)
Thermally agitated carriers in the channel cause a randomly varying current

\[ \overline{i_{nd}^2} \bigg|_{th} = 4kT \gamma g_{do} \Delta f \]

- \( \gamma \) is called excess noise factor
  - = 2/3 in long channel
  - = 2 to 3 (or higher!) in short channel NMOS (less in PMOS)
- \( g_{do} \) will be discussed shortly \( \text{(Note: } g_{do} = g_m / \alpha) \)
**Drain Noise – 1/f (Assume Device in Saturation)**

- Traps at channel/oxide interface randomly capture/release carriers

\[
\bar{i}_{nd}^2 \frac{1}{f} = \frac{K_f}{f^n} \Delta f \approx \frac{K}{f} \frac{g_m^2}{W L C_{ox}^2} \Delta f
\]

- Parameterized by \( K_f \) and \( n \)
  - Provided by fab (note \( n \approx 1 \))
  - Currently: \( K_f \) of PMOS << \( K_f \) of NMOS due to buried channel
- To minimize: want large area (high WL)
**Induced Gate Noise (Assume Device in Saturation)**

- Fluctuating channel potential couples capacitively into the gate terminal, causing a noise gate current

\[
\overline{i_{ng}^2} = 4kT \delta g_{do} \left( \frac{2\pi f}{\sqrt{5}/\alpha (g_m/C_{gs})} \right)^2 \Delta f 4kT \delta g_{do}
\]

- \( \delta \) is gate noise coefficient
  - Typically assumed to be \( 2\gamma \)
  - Correlated to drain noise!

(Note: \( \alpha = g_m/g_{do} \))
Useful References on MOSFET Noise

- **Thermal Noise**

- **Gate Noise**
  - Jung-Suk Goo et. al., “The Equivalence of van der Ziel and BSIM4 Models in Modeling the Induced Gate Noise of MOSFETS”, IEDM 2000, 35.2.1-35.2.4
Drain-Source Conductance: $g_{do}$

- $g_{do}$ is defined as channel resistance with $V_{ds}=0$
  - Transistor in triode, so that
    \[ I_d = \mu_n C_{ox} \frac{W}{L} \left( (V_{gs} - V_T)V_{ds} - \frac{V_{ds}^2}{2} \right) \]
    \[ \Rightarrow g_{do} = \left. \frac{dI_d}{dV_{ds}} \right|_{V_{ds}=0} = \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_T) \]
  - Equals $g_m$ for long channel devices
- Key parameters for $0.18\mu$ NMOS devices
  \[ \mu_n = 327.4 \text{ cm}^2/(V \cdot s) \]
  \[ t_{ox} = 4.1 \times 10^{-9} \text{ m} \quad \epsilon_o = 3.9(8.85 \times 10^{-12}) \text{ F/m} \]
  \[ \Rightarrow \mu_n C_{ox} = \mu_n \frac{\epsilon_o}{t_{ox}} = 275.6 \times 10^{-6} \text{ F/(V \cdot s)} \]
  \[ V_T = 0.48 \text{ V} \]
Plot of $g_m$ and $g_{do}$ versus $V_{gs}$ for 0.18$\mu$ NMOS Device

Transconductances $g_m$ and $g_{do}$ versus Gate Voltage $V_{gs}$

For $V_{gs}$ bias voltages around 1.2 V:

$g_{do} = \mu_n C_{ox} W/L (V_{gs} - V_T)$

$g_m$ (simulated in Hspice):

$\alpha = \frac{g_m}{g_{do}} \approx \frac{1}{2}$
Plot of $g_m$ and $g_{do}$ versus $I_{dens}$ for 0.18µ NMOS Device

Transconductances $g_m$ and $g_{do}$ versus Current Density

$g_{do} = \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_T)$

$g_m$ (simulated in Hspice)

$W = 1.8 \mu m$

$\frac{W}{L} = \frac{1.8 \mu m}{0.18 \mu m}$
**Noise Sources in a CMOS Amplifier**

\[ e_{nG}, e_{nD}, e_{ndeg} : \text{noise sources of external resistors} \]

\[ R_{gpar}, e_{ngpar} : \text{parasitic gate resistance and its noise} \]

\[ i_{ng} : \text{induced gate noise,} \]

\[ g_g : \text{caused by distributed nature of channel} \]

\[ i_{nd} : \text{drain noise (thermal and 1/f)} \]

\[
\begin{align*}
 g_g & = \frac{w^2 C^2_{gs}}{5 g_{d0}} 
\end{align*}
\]
$R_{gpar}, \bar{e}_{ngpar}$: can make negligible with proper layout

$g_{g}$: assume to be negligible (for $w \ll w_t$)

$C_{sb}, C_{gd}, C_{db}, g_{mb}$: too painful to include for calculations

$r_o$: impact is minor since $R_D$ is small (for high bandwidth)
Key Noise Sources for Noise Analysis

- Transistor drain noise
  \[ \overline{i^2_{nd}} = 4kT \gamma g_{do} \Delta f + \frac{Kf}{f_n} \Delta f \]

- Transistor gate noise
  \[ \overline{i^2_{ng}} = 4kT \delta g_g \Delta f \]
  \[ \text{where } g_g = \frac{w^2 C^2_{gs}}{5g_{d0}} \]

\[ \overline{e^2_{nG}} = 4kTR_G \Delta f \]

\[ \overline{e^2_{nD}} = 4kTR_D \Delta f \]

\[ \overline{e^2_{nD}} = 4kTR_{deg} \Delta f \]
Apply Thevenin Techniques to Simplify Noise Analysis

- Assumption: noise independent of load resistor on drain
Calculation of Equivalent Output Noise for Each Case

\[ i_{out} = g_m Z_{gs} i_{ng} + \eta i_{nd} \]

\[ i_{ndg} = g_m Z_{gs} i_{ng} + \eta i_{nd} \]
Calculation of $Z_{gs}$

- **Write KCL equations**
  
  \[
  (1) \quad -i_{test} + \frac{v_{test}}{1/(sC_{gs})} + g_m v_{test} = \frac{v_1}{Z_{deg}}
  \]
  
  \[
  (2) \quad \frac{v_{test}}{Z_g} + \frac{v_1}{Z_{deg}} = g_m v_{test}
  \]

- **After much algebra:**
  
  \[
  Z_{gs} = \frac{v_{test}}{i_{test}} = \frac{1}{sC_{gs}} + \frac{Z_{deg} + Z_g}{1 + g_m Z_{deg}}
  \]
Calculation of $\eta$

- **Determine $V_{gs}$ to find $i_{out}$ in terms of $i_{test}$**

  
  1. $i_{out} = i_{test} + g_m v_{gs}$
  2. $v_{gs} = -v_1 \frac{1/(sC_{gs})}{1/(sC_{gs}) + Z_g}$
  3. $v_1 = i_{out} (Z_{deg} \parallel \left( \frac{1}{sC_{gs}} + Z_g \right))$

- **After much algebra:**

  
  $\eta = \frac{i_{out}}{i_{test}} = 1 - \left( \frac{g_m Z_{deg}}{Z_{deg} + Z_g} \right) Z_{gs}$
Calculation of Output Current Noise Variance (Power)

To find noise variance:

\[ \frac{i_{out}^2}{\overline{\dot{i}_{ndg}^2}} = \frac{\dot{i}_{ndg} \overline{\dot{i}_{ndg}}}{\overline{\dot{i}_{ndg} \overline{\dot{i}_{ndg}}}} = (\eta \dot{i}_{nd} + gmZ_{gs} \overline{\dot{i}_{ng}})(\eta \dot{i}_{nd} + gmZ_{gs} \overline{\dot{i}_{ng}}) \]
\[ i_{ndg}^2 = |\eta|^2 i_{nd}^2 i_{nd}^* + i_{nd}^* i_{ng} g m \eta^* Z_{gs} + i_{nd} i_{ng}^* (g m \eta Z_{gs})^* + i_{ng} i_{ng}^* |g m Z_{gs}|^2 \]

\[ = |\eta|^2 i_{nd}^2 + 2 \text{Re} \left\{ i_{nd}^* i_{ng} g m \eta^* Z_{gs} \right\} + i_{ng}^2 |g m Z_{gs}|^2 \]

\[ = |\eta|^2 i_{nd}^2 + 2 \text{Re} \left\{ \frac{i_{nd}^* i_{ng} g m \eta^* Z_{gs}}{\sqrt{i_{nd}^2 i_{ng}^2}} \right\} + i_{ng}^2 |g m Z_{gs}|^2 \]

**Define correlation coefficient \( c \) between \( i_{ng} \) and \( i_{nd} \)

\[ c = \frac{i_{nd}^* i_{ng}}{\sqrt{i_{nd}^2 i_{ng}^2}} \Rightarrow i_{ndg}^2 = |\eta|^2 i_{nd}^2 + 2 \text{Re} \left\{ c \frac{i_{nd}^2 i_{ng}^2 g m \eta^* Z_{gs}}{i_{nd}^2 i_{nd}^2} \right\} + i_{ng}^2 |g m Z_{gs}|^2 \]

\[ i_{ndg}^2 = i_{nd}^2 \left( |\eta|^2 + 2 \text{Re} \left\{ c \frac{i_{ng}^2 g m \eta^* Z_{gs}}{i_{nd}^2} \right\} + \frac{i_{ng}^2 g m^2 |Z_{gs}|^2}{i_{nd}^2} \right) \]
Parameterized Expression for Output Noise Variance

- Key equation from last slide

\[ \overline{i_{ndg}^2} = \overline{i_{nd}^2} \left( |\eta|^2 + 2\text{Re} \left\{ c \sqrt{\frac{i_{ng}^2}{i_{nd}^2}} g_m \eta^* Z_{gs} \right\} \right) + \frac{i_{ng}^2}{i_{nd}^2} g_m^2 |Z_{gs}|^2 \]

- Solve for noise ratio

\[ \sqrt{\frac{i_{ng}^2}{i_{nd}^2}} g_m = g_m \sqrt{\frac{4kT\delta (wC_{gs})^2 / (5g_{do})}{4kT\gamma g_{do}}} = \frac{g_m}{g_{do}} \sqrt{\frac{\delta}{5\gamma}} (wC_{gs}) \]

- Define parameters \( Z_{gsw} \) and \( \chi_d \)

\[ Z_{gsw} = wC_{gs} Z_{gs}, \quad \chi_d = \frac{g_m}{g_{do}} \sqrt{\frac{\delta}{5\gamma}} \]

\[ \Rightarrow \quad \overline{i_{ndg}^2} = \overline{i_{nd}^2} \left( |\eta|^2 + 2\text{Re} \left\{ c\chi_d \eta^* Z_{gsw} \right\} + \chi_d^2 |Z_{gsw}|^2 \right) \]
Small Signal Model for Noise Calculations

\[
\frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left( |\eta|^2 + 2 \text{Re} \left\{ c\chi_d \eta^* Z_{gs} \right\} + \chi_d^2 |Z_{gs}|^2 \right)
\]

where:
\[
\frac{i_{nd}^2}{\Delta f} = 4kT\gamma g_{do}, \quad \chi_d = \frac{g_m}{g_{do}} \sqrt{\frac{\delta}{5\gamma}}, \quad Z_{gs} = \omega C_{gs} Z_g
\]

\[
Z_{gs} = \frac{1}{sC_{gs}} \left| \frac{Z_{deg} + Z_g}{1 + g_m Z_{deg}} \right|
\]

\[
\eta = 1 - \left( \frac{g_m Z_{deg}}{Z_{deg} + Z_g} \right) Z_{gs}
\]
Example: Output Current Noise with $Z_s = R_s, Z_{deg} = 0$

- **Step 1:** Determine key noise parameters
  - For 0.18µ CMOS, we will assume the following

  \[
  c = -j0.55, \quad \gamma = 3, \quad \delta = 2\gamma = 6, \quad \frac{g_m}{g_{do}} = \frac{1}{2} \quad \Rightarrow \quad \chi_d = 0.32
  \]

- **Step 2:** calculate $\eta$ and $Z_{gsw}$

  \[
  \eta = 1, \quad Z_{gsw} = wC_{gs} \left( R_s \left| \frac{1}{j\omega C_{gs}} \right| \right) = \frac{wC_{gs}R_s}{1 + j\omega C_{gs}R_s}
  \]
Calculation of Output Current Noise (continued)

- **Step 3:** Plug values into the previously derived expression

\[
\frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left(1 + 2 \text{Re} \left\{ -j |\chi_d Z_{gs}w| \right\} + \chi_d^2 |Z_{gs}w|^2 \right)
\]

**Drain Noise Multiplying Factor**

\[
Z_{gs}w = wC_{gs} \left( R_s \left| \frac{1}{jwC_{gs}} \right| \right) = \frac{wC_{gs}R_s}{1 + jwC_{gs}R_s}
\]

- For \( w << 1/(R_s C_{gs}) \):

\[
Z_{gs}w \approx wC_{gs}R_s \quad \Rightarrow \quad \frac{i_{ndg}^2}{\Delta f} \approx \frac{i_{nd}^2}{\Delta f} \left(1 + \chi_d^2 (wC_{gs}R_s)^2 \right)
\]

**Gate noise contribution**
Calculation of Output Current Noise (continued)

- Step 3: Plug values into the previously derived expression

\[
\frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left( 1 + 2 \text{Re} \left\{ -j|c|\chi_d Z_{gsw} \right\} + \chi_d^2 |Z_{gsw}|^2 \right)
\]

Drain Noise Multiplying Factor

\[
Z_{gsw} = wC_{gs} \left( R_s \left| \frac{1}{jwC_{gs}} \right| \right) = \frac{wC_{gs}R_s}{1 + jwC_{gs}R_s}
\]

- For \( w >> 1/(R_sC_{gs}) \):

\[
Z_{gsw} \approx \frac{1}{j} \quad \Rightarrow \quad \frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left( 1 - 2|c|\chi_d + \chi_d^2 \right)
\]

Gate noise contribution
Conclusion: gate noise has little effect on common source amp when source impedance is purely resistive!
Drain thermal noise is the chief issue of concern when designing amplifiers with > 1 GHz bandwidth
- 1/f noise corner is usually less than 1 MHz
- Gate noise contribution only has influence at high frequencies

Noise performance specification is usually given in terms of input referred voltage noise
Narrowband Amplifier Noise Requirements

- Here we focus on a narrowband of operation
  - Don’t care about noise outside that band since it will be filtered out
- Gate noise is a significant issue here
  - Using reactive elements in the source dramatically impacts the influence of gate noise
- Specification usually given in terms of Noise Figure

\[
\frac{2}{\text{Indg}} \frac{\Delta f}{\Delta f} = \frac{4kT\gamma g_{do}}{f} \\
\text{drain 1/f noise} \\
\text{drain thermal noise} \\
\text{1/f noise corner frequency} \\
\text{Narrowband amplifier frequency range} \\
\frac{1}{2\pi R_s C_{gs}}
\]
The Impact of Gate Noise with $Z_s = R_s + sL_g$

- **Step 1:** Determine key noise parameters
  - For 0.18μ CMOS, again assume the following

$$c = -j0.55, \quad \gamma = 3, \quad \delta = 2\gamma = 6, \quad \frac{g_m}{g_{do}} = \frac{1}{2} \quad \Rightarrow \quad \chi_d = 0.32$$

- **Step 2:** Note that $\eta = 1$, calculate $Z_{gs,w}$

$$Z_{gs,w} = wC_{gs} \left( (R_s + j\omega L_g) || \frac{1}{j\omega C_{gs}} \right) = \frac{wC_{gs}(R_s + j\omega L_g)}{1 - \omega^2 L_g C_{gs} + j\omega C_{gs} R_s}$$
Evaluate $Z_{gsw}$ At Resonance

- Set $L_g$ such that it resonates with $C_{gs}$ at the center frequency ($w_o$) of the narrow band of interest

$$\Rightarrow \frac{1}{\sqrt{L_g C_{gs}}} = w_o$$

Note: $Q = \frac{1}{w_o C_{gs} R_s} = \frac{w_o L_g}{R_s}$

- Calculate $Z_{gsw}$ at frequency $w_o$

$$Z_{gsw} = \frac{w_o C_{gs}(R_s + j w_o L_g)}{1 - w_o^2 L_g C_{gs} + j w_o C_{gs} R_s} = w_o C_{gs}(Q^2 R_s - j \sqrt{L_g/C_{gs}})$$

$$= Q - j$$
The Impact of Gate Noise with \( Z_s = R_s + sL_g \) (Cont.)

- **Key noise expression derived earlier**

\[
\frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left( 1 + 2 \Re \{ -jc|\chi_d Z_{gsW} \} + \chi_d^2 |Z_{gsW}|^2 \right)
\]

- **Substitute in for \( Z_{gsW} \)**

\[
2 \Re \{ -jc|\chi_d Z_{gsW} \} = 2 \Re \{ -jc|\chi_d (Q - j) \} = -2jc|\chi_d
\]

\[
\chi_d^2 |Z_{gsW}|^2 = \chi_d^2 |Q - j|^2 = \chi_d^2 (Q^2 + 1)
\]

\[
\Rightarrow \quad \frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left( 1 - 2jc|\chi_d + \chi_d^2 (Q^2 + 1) \right)
\]

**Gate noise contribution**

- **Gate noise contribution is a function of \( Q \)**
  - Rises monotonically with \( Q \)
At What Value of Q Does Gate Noise Exceed Drain Noise?

- Determine crossover point for Q value

\[
\frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left( 1 - 2|c|\chi_d + \chi_d^2(Q^2 + 1) \right) = \frac{i_{nd}^2}{\Delta f} 1
\]

\[\Rightarrow Q = \sqrt{1/\chi_d^2 - 1 + 2|c|/\chi_d} \quad (= 3.5 \text{ for } 0.18\mu \text{ specs})\]

- Critical Q value for crossover is primarily set by process
Calculation of the Signal Spectrum at the Output

- First calculate relationship between $v_{\text{in}}$ and $i_{\text{out}}$

$$i_{\text{out},\text{sig}} = g_m v_{gs} = g_m \frac{1}{1 - w^2 L_g C_{gs} + jw R_s C_{gs}} V_{\text{in}}$$

- At resonance:

$$i_{\text{out},\text{sig}} = g_m v_{gs} = g_m \frac{1}{jw_0 R_s C_{gs}} v_{\text{in}} = g_m (-jQ)v_{\text{in}}$$

- Spectral density of signal at output at resonant frequency

$$S_{i_{\text{out},\text{sig}}}(f) = |g_m (-jQ)|^2 S_{\text{in}}(f) = (g_m Q)^2 S_{\text{in}}(f)$$
Impact of Q on SNR (Ignoring $R_s$ Noise)

- **SNR (assume constant spectra, ignore noise from $R_s$):**

  $SNR_{out} = \frac{S_{iout, sig}(f)}{S_{iout, noise}(f)} \approx \frac{(g_m Q)^2 S_{in}(f)}{i_{ndg}^2 / \Delta f}$

- For small Q such that gate noise < drain noise
  - $SNR_{out}$ improves dramatically as Q is increased

- For large Q such that gate noise > drain noise
  - $SNR_{out}$ improves very little as Q is increased
Noise Factor and Noise Figure

- **Definitions**

  Noise Factor \( F = \frac{SNR_{in}}{SNR_{out}} \)

  Noise Figure \( = 10 \log(\text{Noise Factor}) \)

- **Calculation of \( SNR_{in} \) and \( SNR_{out} \)**

  \[
  SNR_{in} = \frac{|\alpha|^2 v_{in}^2}{|\alpha|^2 e_{nRs}^2} = \frac{v_{in}^2}{e_{nRs}^2} \quad \text{where} \quad \alpha = \frac{Z_{in}}{R_s + Z_{in}}
  \]

  \[
  SNR_{out} = \frac{|\alpha|^2 |G_m|^2 v_{in}^2}{|\alpha|^2 |G_m|^2 e_{nRs}^2 + i_{nout}^2} \quad \text{where} \quad G_m = \frac{i_{out}}{v_x}
  \]
Calculate Noise Factor (Part 1)

- First calculate $SNR_{out}$ (must include $R_s$ noise for this)

  - $R_s$ noise calculation (same as for $V_{in}$)

  \[
  i_{out,Rs} = g_m (-jQ) \overline{e_{ns}} \Rightarrow S_{i_{out,Rs}}(f) = (g_m Q)^2 4kT R_s
  \]

  - $SNR_{out}$:

  \[
  SNR_{out} = \frac{(g_m Q)^2 S_{in}(f)}{i_{ndg}^2/\Delta f + (g_m Q)^2 4kT R_s}
  \]

- Then calculate $SNR_{in}$:

\[
SNR_{in} = \frac{S_{in}(f)}{e_{ns}^2/\Delta f} = \frac{S_{in}(f)}{4kT R_s}
\]
Calculate Noise Factor (Part 2)

\[ SNR_{\text{out}} = \frac{|g_mQ|^2 S_{\text{in}}(f)}{i_{\text{ndg}}^2/\Delta f + (g_mQ)^2 4kTR_s} \]

\[ SNR_{\text{in}} = \frac{S_{\text{in}}(f)}{e_{\text{ns}}^2/\Delta f} = \frac{S_{\text{in}}(f)}{4kTR_s} \]

- **Noise Factor calculation:**

\[
\text{Noise Factor} = \frac{SNR_{\text{in}}}{SNR_{\text{out}}} = \frac{i_{\text{ndg}}^2/\Delta f + |g_mQ|^2 4kTR_s}{(g_mQ)^2 4kTR_s}
\]

\[
= 1 + \frac{i_{\text{ndg}}^2/\Delta f}{(g_mQ)^2 4kTR_s}
\]

- **From previous analysis**

\[
\overline{i_{\text{ndg}}^2/\Delta f} = 4kT \gamma g_{do} \left( 1 - 2|c| \chi_d + (Q^2 + 1) \chi_d^2 \right)
\]

\[ \Rightarrow \text{Noise Factor} = 1 + \frac{\gamma g_{do} \left( 1 - 2|c| \chi_d + (Q^2 + 1) \chi_d^2 \right)}{(g_mQ)^2 R_s} \]
Calculate Noise Factor (Part 3)

Noise Factor = \[ 1 + \frac{\gamma g_{do} \left( 1 - 2|c|\chi_d + (Q^2 + 1)\chi_d^2 \right)}{(g_m Q)^2 R_s} \]

- Modify denominator using expressions for Q and \( w_t \)

\[ Q = \frac{1}{w_o R_s C_{gs}}, \quad w_t \approx \frac{g_m}{C_{gs}} \]

\[ \Rightarrow (g_m Q)^2 R_s = g_m^2 Q \frac{R_s}{w_o R_s C_{gs}} = g_m Q \frac{g_m}{C_{gs} w_o} = g_m Q \frac{w_t}{w_o} \]

- Resulting expression for noise factor:

\[ \text{Noise Factor} = 1 + \left( \frac{w_o}{w_t} \right) \gamma \left( \frac{g_{do}}{g_m} \right) \frac{1}{Q} \left( 1 - 2|c|\chi_d + (Q^2 + 1)\chi_d^2 \right) \]

Noise Factor scaling coefficient

- Noise factor primarily depends on Q, \( w_o/w_t \), and process specs
Minimum Noise Factor

\[
\text{Noise Factor} = 1 + \left( \frac{w_o}{w_t} \right) \gamma \left( \frac{g_{do}}{g_m} \right) \frac{1}{Q} \left( 1 - 2|c|\chi_d + (Q^2 + 1)\chi_d^2 \right)
\]

**Noise Factor scaling coefficient**

- We see that the noise factor will be minimized for some value of Q
  - Could solve analytically by differentiating with respect to Q and solving for peak value (i.e. where deriv. = 0)
- In Tom Lee’s book (pp 272-277), the minimum noise factor for the MOS common source amplifier (i.e. no degeneration) is found to be:

\[
\text{Min Noise Factor} = 1 + \left( \frac{w_o}{w_t} \right) \frac{2}{\sqrt{5}} \sqrt{\gamma\delta\left(1 - |c|^2\right)}
\]

**Noise Factor scaling coefficient**

- How do these compare?
Plot of Minimum Noise Factor and Noise Factor Vs. Q

Achievable values as a function of Q under the constraint that

\[
\frac{1}{\sqrt{L_gC_{gs}}} = w_0
\]

Minimum across all values of Q and

\[
\frac{1}{\sqrt{L_gC_{gs}}}
\]

Note: curves meet if we approximate \(Q^2 + 1 \approx Q^2\)
Achieving Minimum Noise Factor

- For common source amplifier without degeneration
  - Minimum noise factor can only be achieved at resonance if gate noise is uncorrelated to drain noise (i.e., if c = 0) – we’ll see this next lecture
  - We typically must operate slightly away from resonance in practice to achieve minimum noise factor since c will be nonzero

- How do we determine the optimum source impedance to minimize noise figure in classical analysis?
  - Next lecture!