

Analysis and Design of Analog Integrated Circuits
Lecture 9

Open Circuit Time Constant Technique

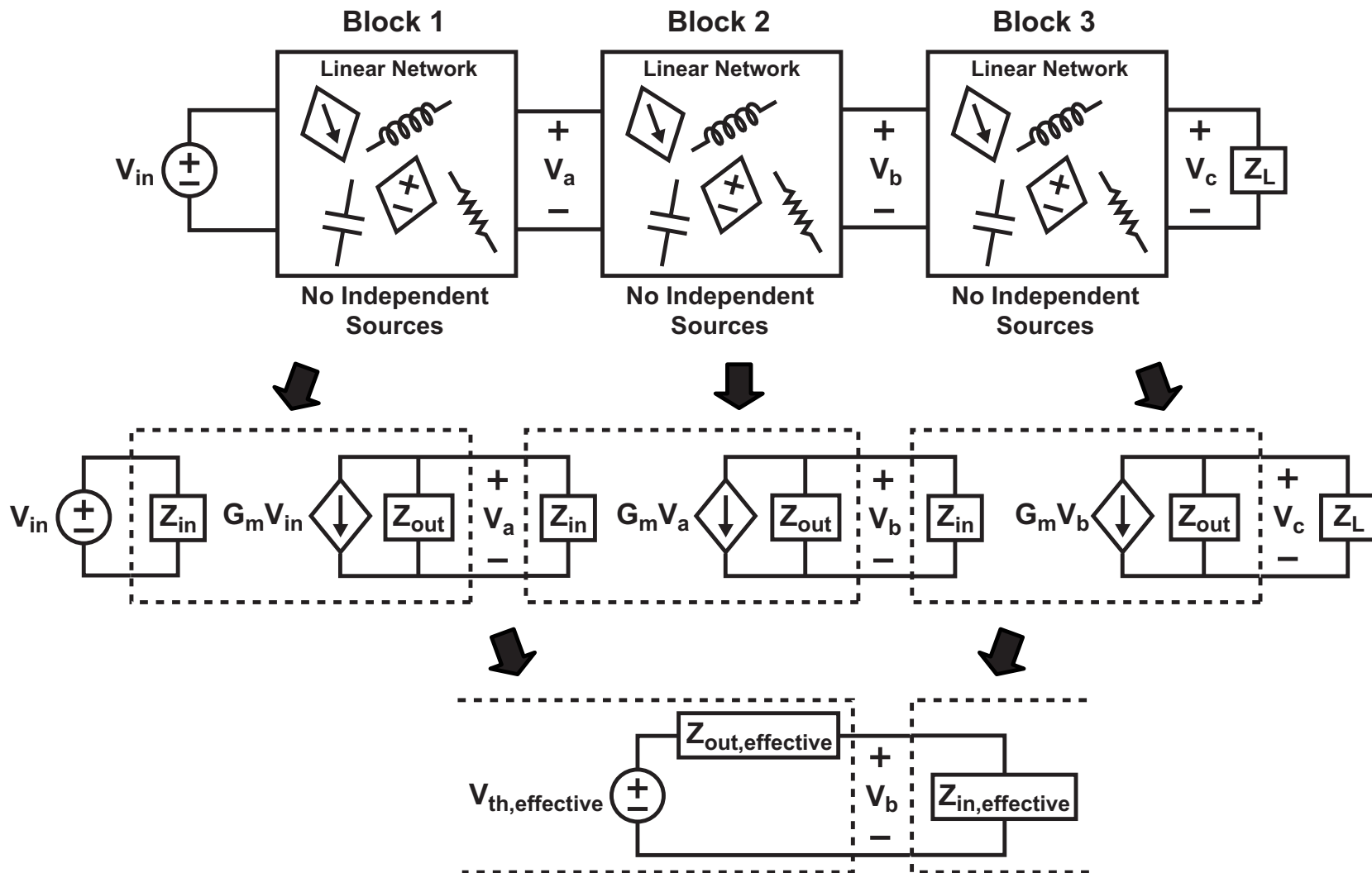
Michael H. Perrott

February 27, 2011

Copyright © 2011 by Michael H. Perrott

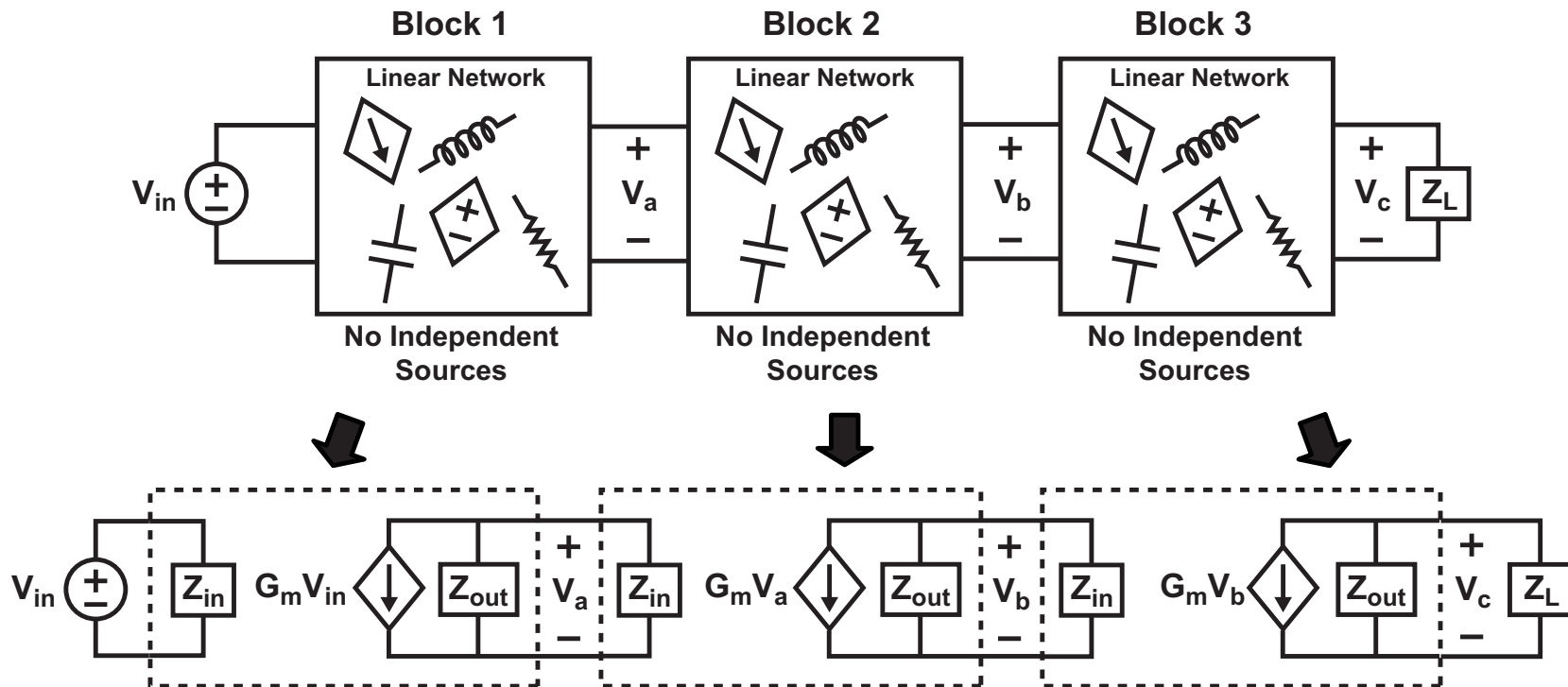
All rights reserved.

Review of Our Analysis Techniques



- Two port analysis allows us to quickly calculate small signal gain from cascaded network stages
 - So far, only purely resistive impedances have been considered

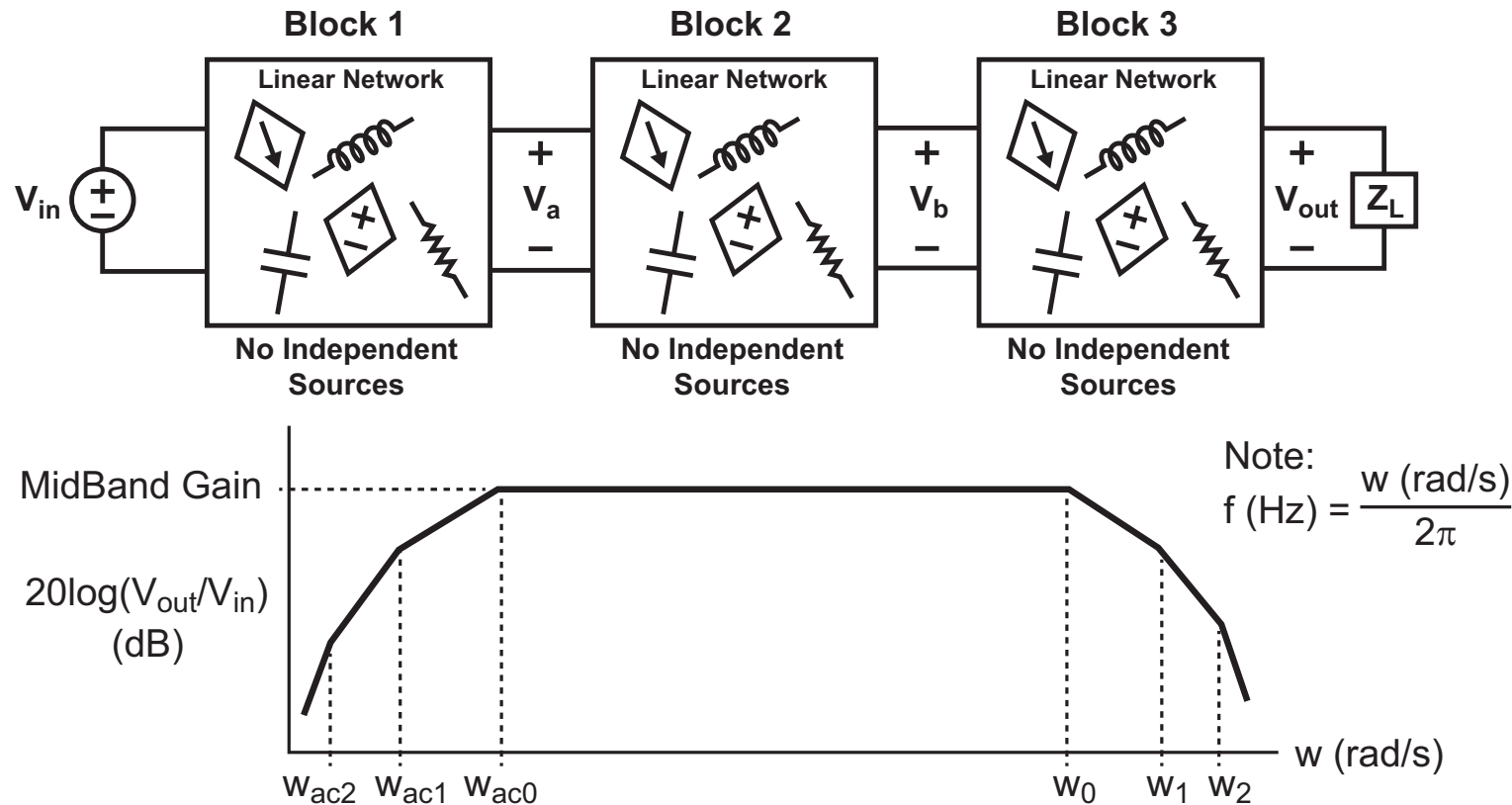
The Problem with Complex Impedances



- When complex impedances are considered (i.e., capacitors, inductors, and resistors), things get much more messy
 - Complex impedance calculations are time consuming
 - Capacitance between drain and gate of transistors complicates calculation effort further

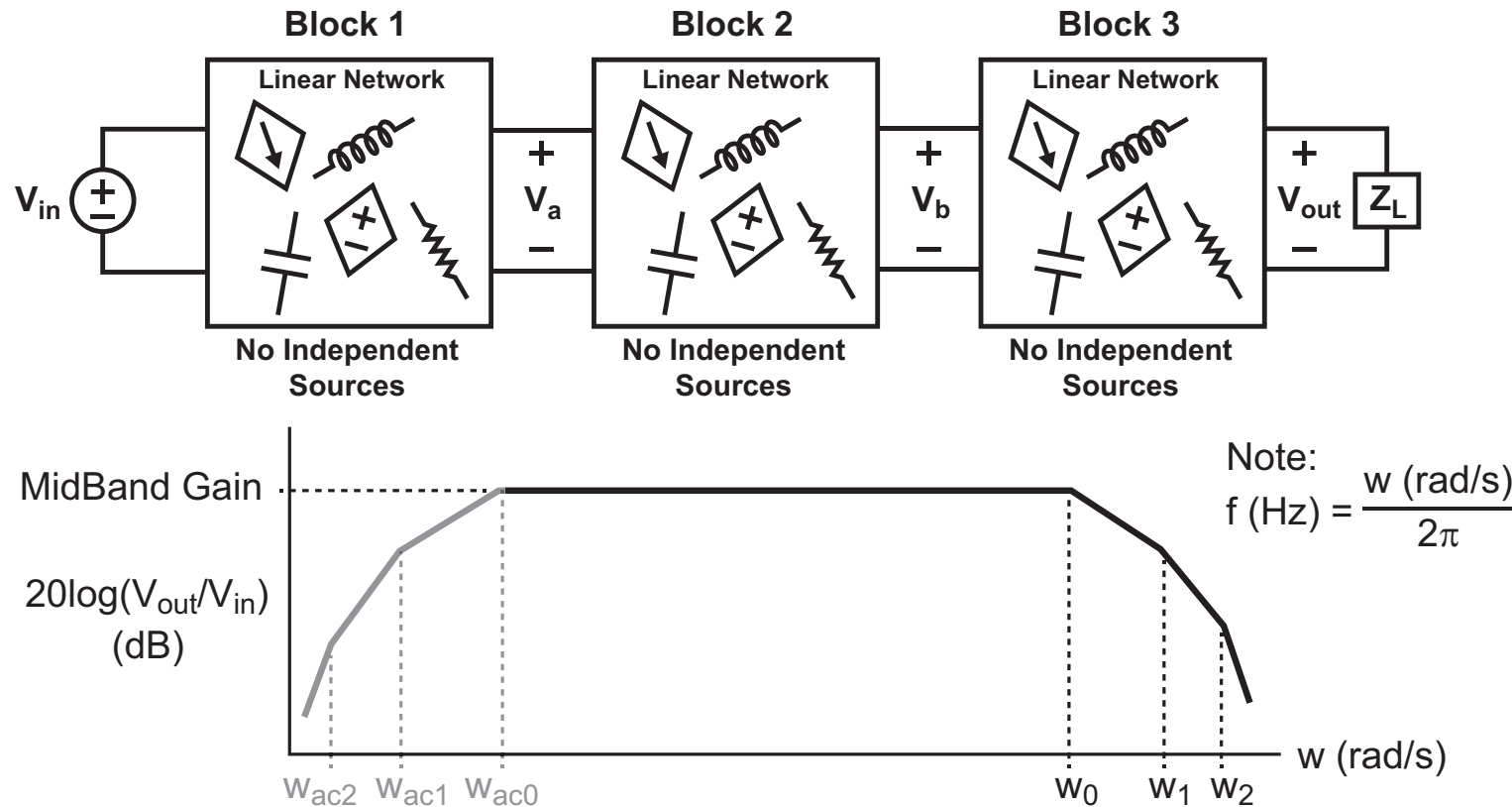
Can we determine a faster analysis path to gain intuition?

General Frequency Response for Amplifiers



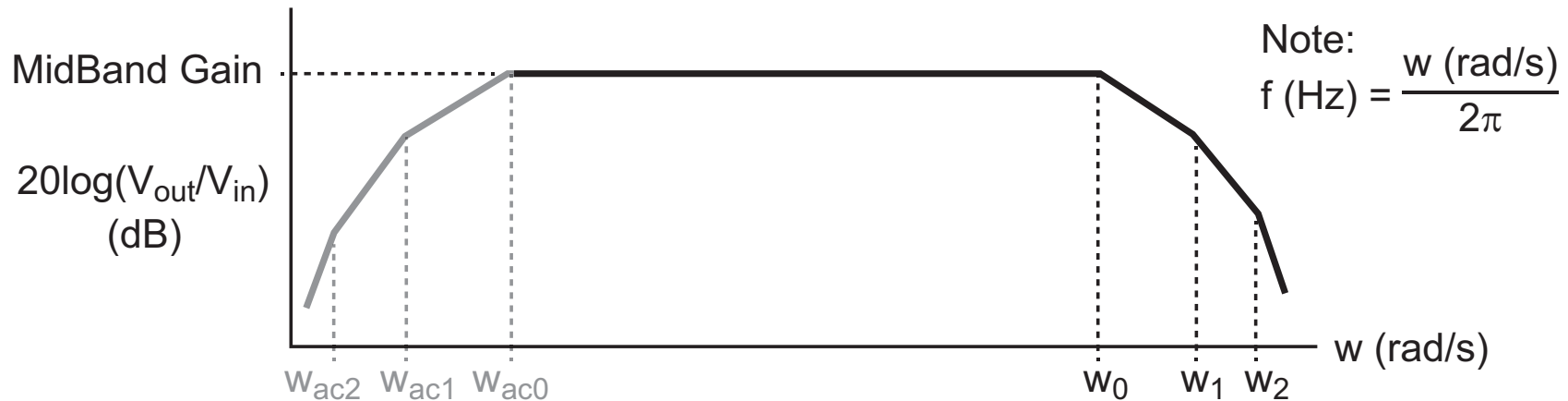
- Midband gain can be calculated by assuming purely resistive impedances (as we have done so far)
 - Large valued capacitors used for AC coupling will be shorts in this analysis
 - For DC coupled circuits, typically DC gain = Midband Gain
 - Small valued capacitors will be opens in this analysis

Our Focus Will Be on High Frequency Poles



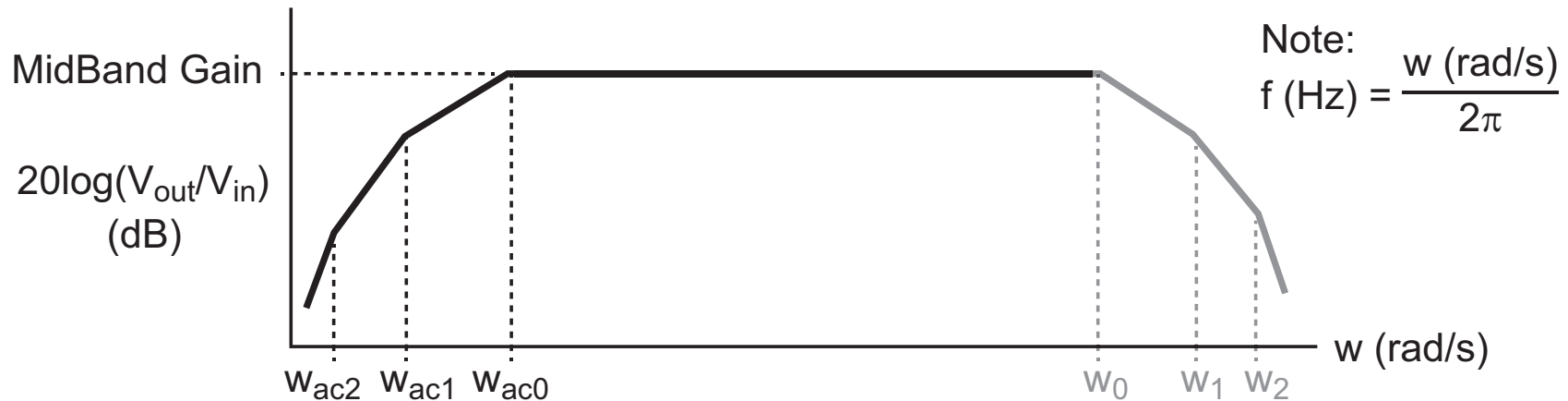
- We are particularly interested in knowing the bandwidth of our amplifier circuit
 - Bandwidth is primarily set by the lowest frequency pole, ω_0
 - Additional attenuation occurs at frequencies beyond the amplifier bandwidth by higher frequency poles ω_1 , ω_2 , etc.

Open Circuit Time Constant Technique



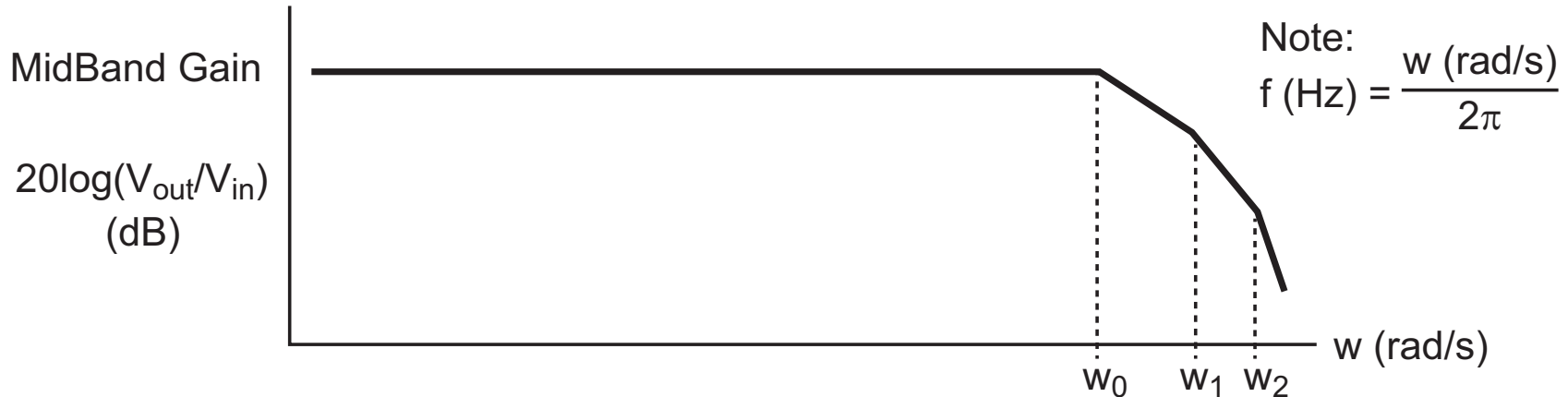
- **The Open Circuit Time Constant (OCT) technique allows us to quickly estimate the bandwidth of an amplifier circuit**
 - We will see that it is most accurate when there is one *dominant pole*, w_0
 - This means that w_1 , w_2 , and higher poles are not close in frequency to w_0
 - This will hold for opamps and other circuits that operate in feedback
 - There is still considerable value to the OCT method in providing design intuition even when there is not just one dominant pole

Short Circuit Time Constant Technique



- **The Short Circuit Time Constant (SCT) technique allows us to quickly estimate the AC-coupled cutoff frequency, W_{ac0}**
 - This has many similarities to the OCT method, but we will not discuss in this class since
 - AC coupling is not used very often in integrated circuits due to the high cost of large valued capacitors
 - When AC coupling is applied in integrated circuits, it is often quite easy to estimate the AC-coupled cutoff frequency since there are relatively few poles in the circuit related to AC-coupling

Key Assumptions for the OCT Technique

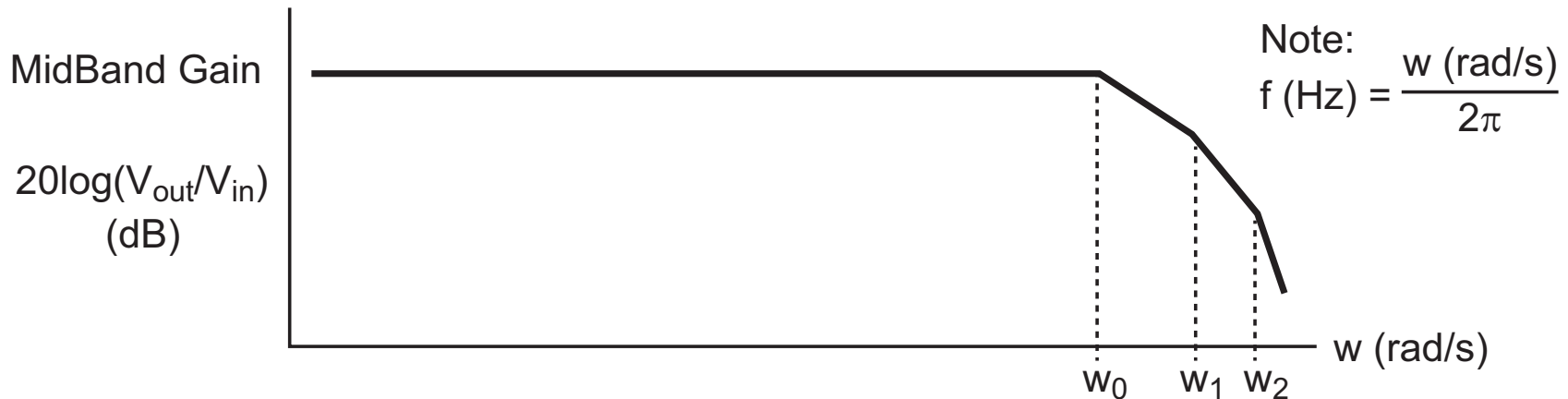


- Let us assume that the transfer function from V_{in} to V_{out} is

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{K}{(\tau_0 s + 1)(\tau_1 s + 1) \cdots (\tau_{n-1} s + 1)}$$

- Note that we are ignoring any AC-coupling poles/zeros
 - This implies that we are approximating DC gain = Midband gain
 - The OCT method does not require this assumption – it just simplifies the analysis to follow
- Note also that DC gain equals K in the above transfer function
 - We see this by setting $s = 0$

Key Idea of the OCT Technique



- Assuming the transfer function from V_{in} to V_{out} is:

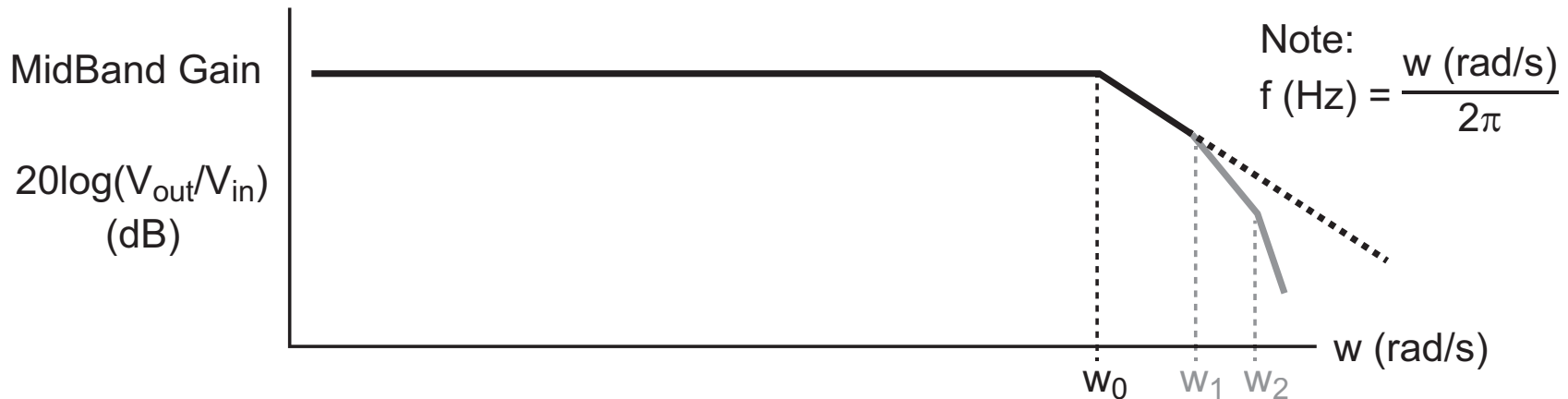
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{K}{(\tau_0 s + 1)(\tau_1 s + 1) \cdots (\tau_{n-1} s + 1)}$$

- We can achieve a reasonable approximation of the bandwidth of the system by instead considering:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{K}{\left(\sum_{i=0}^{n-1} \tau_i\right) s + 1}$$

- Here τ_i are the “time constants” corresponding to the poles of the circuit network

Bandwidth Estimate from OCT Technique



- The OCT technique approximates the transfer function as:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{K}{\left(\sum_{i=0}^{n-1} \tau_i\right) s + 1}$$

- The estimated bandwidth is found by substituting $s = jw_0$ and solving for w_0 such that the magnitude = $K/\sqrt{2}$

$$w_0 = \frac{1}{\sum_{i=0}^{n-1} \tau_i} \Rightarrow \left| \frac{V_{out}(w_0)}{V_{in}(w_0)} \right| = \left| \frac{K}{j1 + 1} \right| = \frac{K}{\sqrt{2}}$$

Bandwidth estimate found by inverting the sum of time constants!

Why Is This Approximation Reasonable?

- Consider a second order example:

$$\frac{V_{out}(w_0)}{V_{in}(w_0)} = \frac{K}{(j\tau_0 w_0 + 1)(j\tau_1 w_0 + 1)}$$

- Expanding:

$$\frac{V_{out}(w_0)}{V_{in}(w_0)} = \frac{K}{-\tau_0\tau_1 w_0^2 + j(\tau_0 + \tau_1)w_0 + 1}$$

- But notice (since the time constant values are > 0):

$$j(\tau_0 + \tau_1)w_0 = j1 \Rightarrow \tau_0 w_0 < 1, \tau_1 w_0 < 1$$

- In fact: $\tau_0\tau_1 w_0^2 \leq 0.25$
- The worse case of $\tau_0\tau_1 w_0^2 = 0.25$ occurs when $\tau_0 = \tau_1$:

$$\left| \frac{V_{out}(w_0)}{V_{in}(w_0)} \right| = \left| \frac{K}{j1 + 1 - 0.25} \right| = \frac{K}{\sqrt{1.56}} \approx \frac{K}{\sqrt{2}}$$

- The approximation will be better for $\tau_0 \neq \tau_1$

Key Issues For the OCT Approximation

- For the higher order transfer function

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{K}{(\tau_0 s + 1)(\tau_1 s + 1) \cdots (\tau_{n-1} s + 1)}$$

- The OCT approximation for bandwidth is

$$BW \approx \frac{1}{\sum_{i=0}^{n-1} \tau_i} \text{ rad/s}$$

- As hinted at by our second order example:
 - The OCT approximation will have much better accuracy if the time constants are different, and particularly if there is one dominant time constant
 - The bandwidth estimate by the OCT method is typically conservative (i.e., actual bandwidth > OCT estimate)
 - Complex poles can lead to actual bandwidth < OCT estimate

But how do we compute $\sum_{i=0}^{n-1} \tau_i$?

OCT Method of Calculating the Sum of Time Constants

- OCT method calculates $\sum_{i=0}^{n-1} \tau_i$ by the following steps:
 - Compute the effective resistance R_{thj} seen by each capacitor, C_j , with other caps as open circuits
 - AC coupling caps are not included – considered as shorts
 - Form the “open circuit” time constant $T_j = R_{thj}C_j$ for each capacitor C_j
 - Sum all of the “open circuit” time constants
- As proved by Richard Adler at MIT

$$\sum_{i=0}^{n-1} \tau_i = \sum_{j=1}^m R_{thj}C_j$$

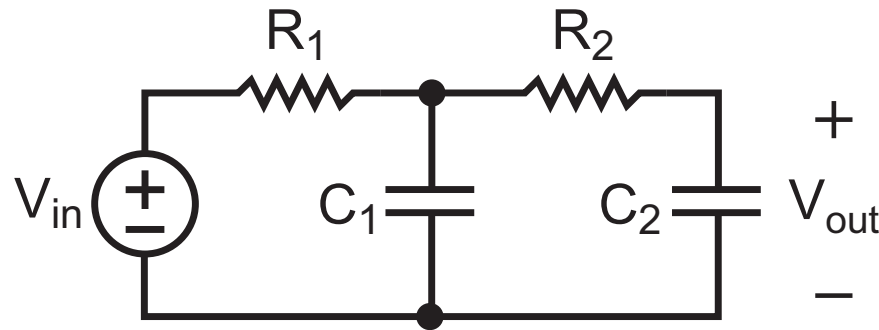
- This implies that the sum of the transfer function pole time constants is the same as the sum of the open circuit time constants

$$\Rightarrow BW \approx \frac{1}{\sum_{j=1}^m R_{thj}C_j} \text{ rad/s}$$

How Do You Tell if a Cap is for AC coupling or OCT?

- In general, capacitors associated with AC coupling have the property that the amplifier gain *increases* as the capacitor goes from open to short
 - These capacitors are simply assumed to be shorts for the OCT analysis
- In general, capacitors used in the OCT calculation have the property that the amplifier gain *decreases* as the capacitor goes from open to short
 - These capacitors must all be considered in the OCT analysis

Example: Second Order RC Network



- **Transfer function of the above network:**

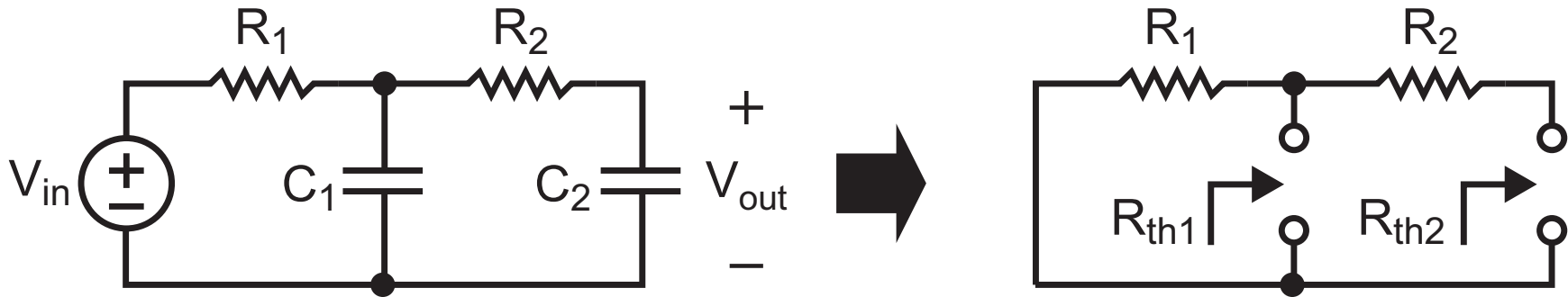
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2) s + 1}$$

- **The sum of the time constants from the poles of the above network are obtained by inspection of the first order coefficient in the above transfer function**

$$\Rightarrow BW \approx \frac{1}{\sum_{i=0}^{n-1} \tau_i} = \frac{1}{R_1 C_1 + R_1 C_2 + R_2 C_2} \text{ rad/s}$$

- **For more complex networks, the direct approach of explicitly calculating the transfer function is quite tedious**

OCT Method Applied to Second Order RC Network



- Obtain the Thevenin resistance values seen by each capacitor with other capacitors as opens

$$R_{th1} = R_1 \Rightarrow R_{th1}C_1 = R_1C_1$$

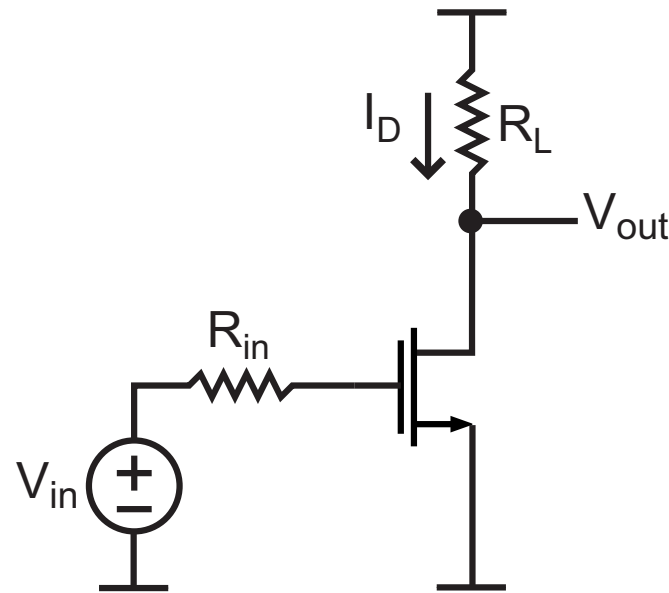
$$R_{th2} = R_1 + R_2 \Rightarrow R_{th2}C_2 = (R_1 + R_2)C_2$$

- Bandwidth estimate from OCT method:

$$\Rightarrow BW \approx \frac{1}{\sum_{j=1}^m R_{thj}C_j} = \frac{1}{R_1C_1 + (R_1 + R_2)C_2} \text{ rad/s}$$

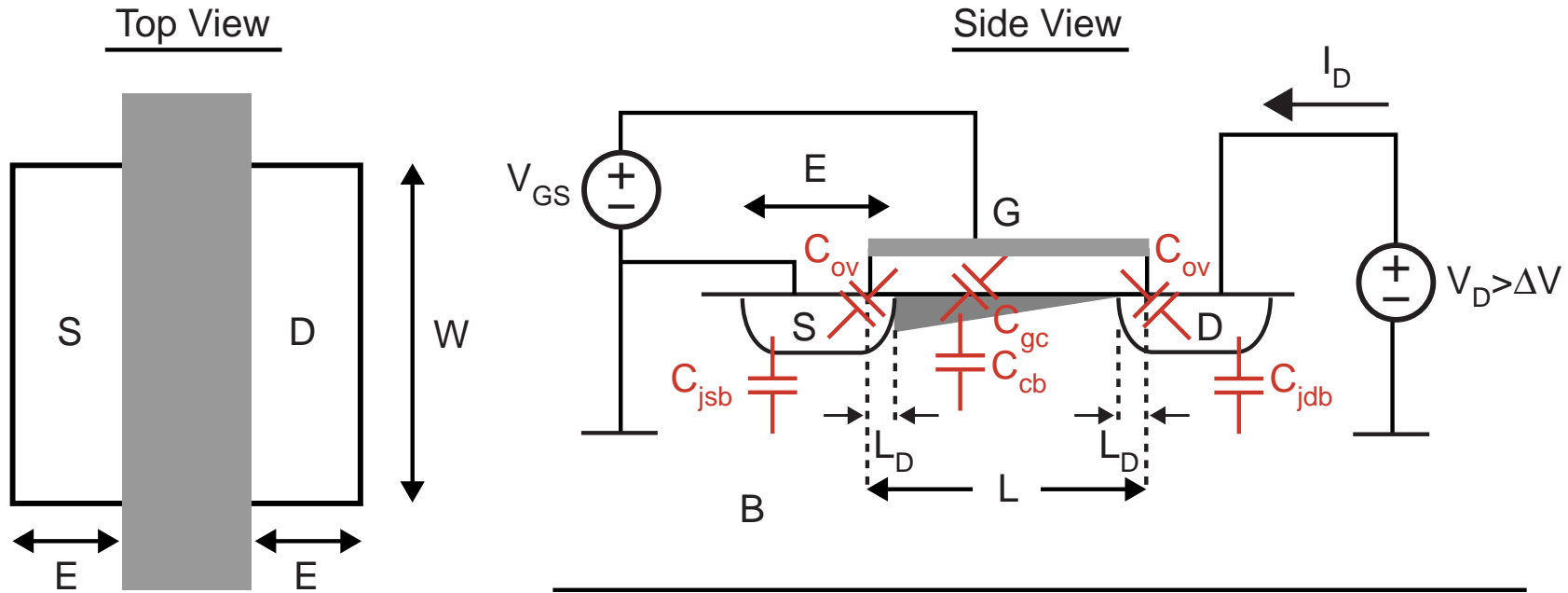
- Note that OCT method agrees with estimate based on direct calculation of the transfer function, but is much faster!

Example: Common Source Amplifier



- Estimate the bandwidth of the above amplifier using the OCT method
 - What capacitances should be considered?
 - What Thevenin resistances must be calculated?

Key Capacitances for CMOS Devices



junction bottom wall
cap (per area)

junction sidewall
cap (per length)

source to bulk cap: $C_{\text{j\text{sb}}} = \frac{C_{\text{j}}(0)}{\sqrt{1 + V_{\text{SB}}/|\Phi_{\text{B}}|}} WE + \frac{C_{\text{j\text{sw}}}(0)}{\sqrt{1 + V_{\text{SB}}/|\Phi_{\text{B}}|}} (W + 2E)$

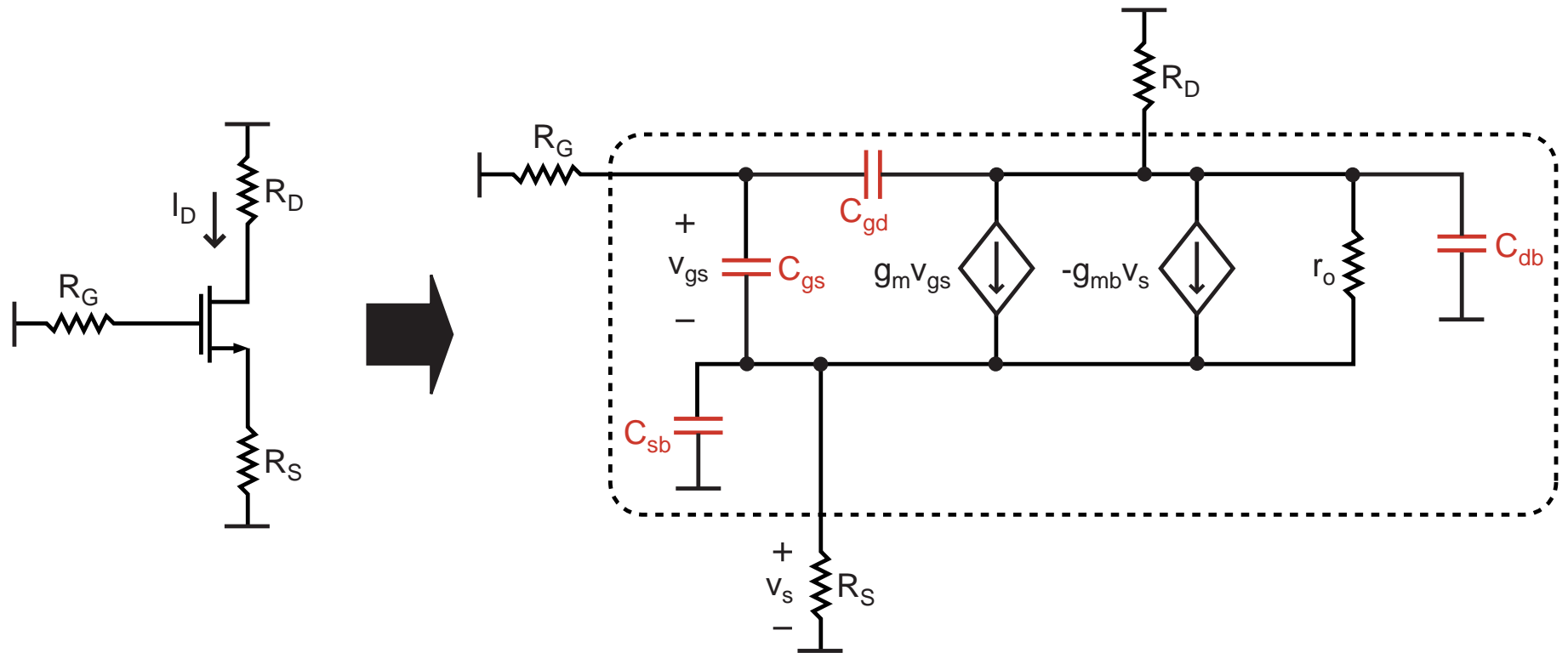
drain to bulk cap: $C_{\text{j\text{sd}}} = \frac{C_{\text{j}}(0)}{\sqrt{1 + V_{\text{DB}}/|\Phi_{\text{B}}|}} WE + \frac{C_{\text{j\text{sw}}}(0)}{\sqrt{1 + V_{\text{DB}}/|\Phi_{\text{B}}|}} (W + 2E)$

(make $2W$ for "4 sided" perimeter in some cases)

overlap cap: $C_{\text{ov}} = WL_D C_{\text{ox}} + WC_{\text{fringe}}$ gate to channel cap: $C_{\text{gc}} = \frac{2}{3} C_{\text{ox}} W(L - 2L_D)$

channel to bulk cap: C_{cb} - ignore in this class

CMOS Hybrid- π Model with Caps (Device in Saturation)



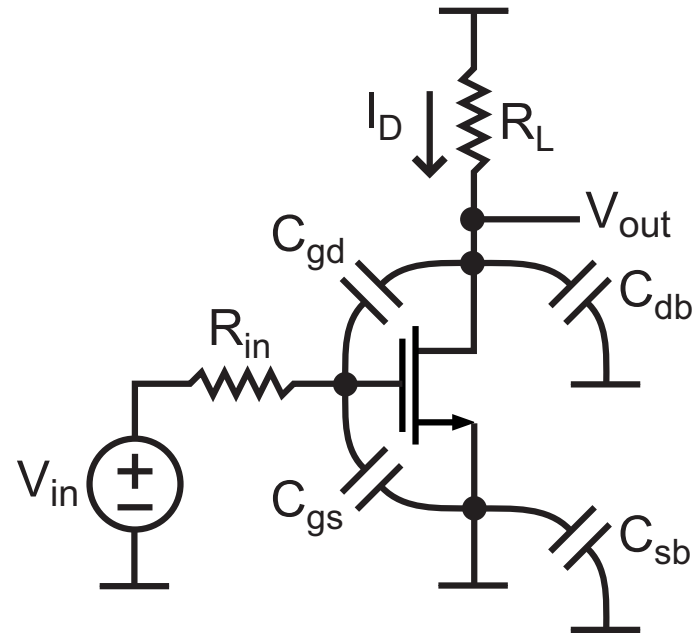
$$C_{gs} = C_{gc} + C_{ov} = \frac{2}{3} C_{ox} W(L-2L_D) + C_{ov}$$

$$C_{gd} = C_{ov}$$

$$C_{sb} = C_{jsb} \quad (\text{area + perimeter junction capacitance})$$

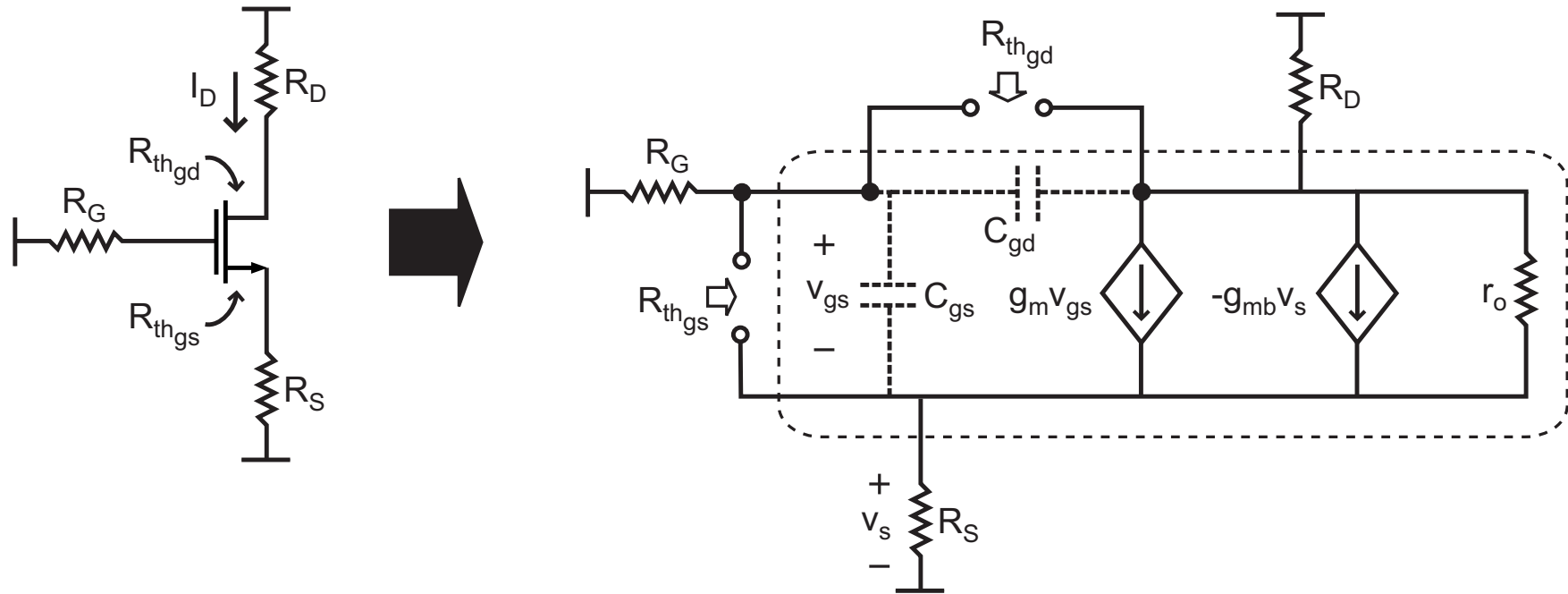
$$C_{db} = C_{jdb} \quad (\text{area + perimeter junction capacitance})$$

Back to Common Source Amplifier



- Of the above capacitors, only C_{gs} , C_{gd} , and C_{db} must be considered
 - C_{sb} is grounded on both sides
- Thevenin resistance calculations
 - C_{db} : $R_{thd} \parallel R_d$
 - C_{gs} and C_{gd} : these involve new Thevenin resistance calculations

OCT Thevenin Resistance Calculations



- C_{gs} : Thevenin resistance between gate and source

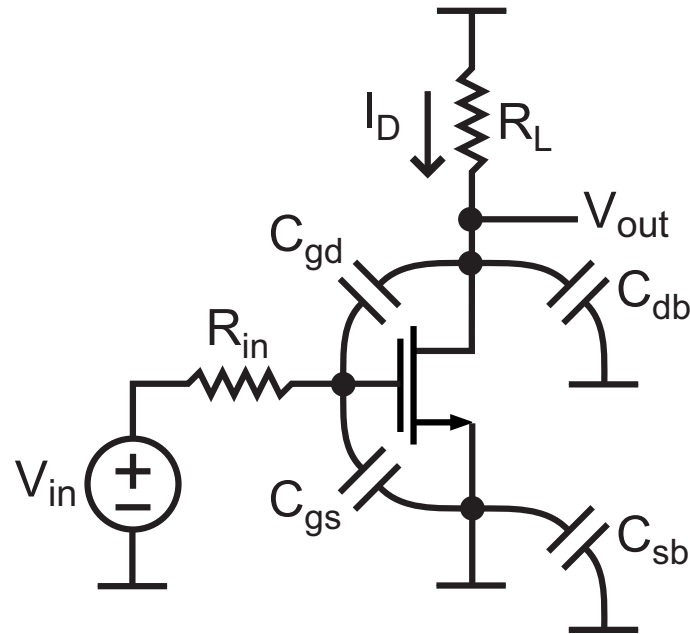
$$R_{th_{gs}} = \frac{R_S(1 + R_D/r_o) + R_G(1 + (g_{mb} + 1/r_o)R_S + R_D/r_o)}{1 + (g_m + g_{mb})R_S + (R_S + R_D)/r_o}$$

- C_{gd} : Thevenin resistance between gate and drain

$$R_{th_{gd}} = (R_D + R_G)(1 - r_{ods}/r_o) + r_{ods}g_m R_G$$

$$\text{where } r_{ods} = r_o \parallel \frac{R_D}{1 + (g_m + g_{mb})R_S}$$

OCT Calculations for Common Source Amplifier



- Estimated bandwidth from OCT method:

$$BW \approx \frac{1}{\sum_{j=1}^m R_{thj} C_j} = \frac{1}{(R_{th_d} || R_d) C_{db} + R_{th_{gd}} C_{gd} + R_{th_{gs}} C_{gs}} \text{ rad/s}$$

- The above calculations are straightforward given the Thevenin resistance formulas for R_{th_d} , $R_{th_{gd}}$, and $R_{th_{gs}}$