Review of Large Signal Analysis of Current Mirrors

\[ \frac{I_2}{I_1} = \frac{1}{2} \mu_n C_{ox} \frac{W_2}{L_2} \left( \frac{\Delta V_2}{(V_{GS2} - V_{TH})^2 (1 + \lambda_2 V_{ds2})} \right) \]

\[ \frac{I_2}{I_1} = \frac{1}{2} \mu_n C_{ox} \frac{W_1}{L_1} \left( \frac{\Delta V_1}{(V_{GS1} - V_{TH})^2 (1 + \lambda_1 V_{ds1})} \right) \]

But, \( V_{TH} + \Delta V_1 = V_{TH} + \Delta V_2 \) \( \Rightarrow \Delta V_1 = \Delta V_2 \)

\[ \frac{I_2}{I_1} = \frac{W_2}{W_1} \frac{L_1}{L_2} \frac{(1 + \lambda_2 V_{ds2})}{(1 + \lambda_1 V_{ds1})} \]

Mismatch due to \( V_{ds} \) difference based on geometry

Note: for accurate ratio, set \( L_1 = L_2 \)
The Issue of $V_{ds}$ Mismatch in Current Mirrors

- **Issue:** Current $I_2$ can vary significantly as a function of the drain voltage of $M_2$
  - We often want a tightly controlled current set only by $I_1$ and transistor sizes
- **How do we improve the current mirror matching performance?**

\[
\frac{I_2}{I_1} = \frac{W_2}{W_1} \frac{(1+\lambda_2 V_{ds2})}{(1+\lambda_1 V_{ds1})}
\]

Mismatch due to $V_{ds}$ difference

Current setting based on geometry

Note: we are assuming $L_1 = L_2$
Cascoded Current Source

- Offers increased output resistance
  - Reduces small signal dependence of output current on the output voltage of the current source
  - From Lecture 6, we derived:
    \[ R_{th_{d3}} \approx r_{o3}(1 + g_{m3}R_{th_{d1}}) = r_{o3}(1 + g_{m3}r_{o1}) \approx (g_{m3}r_{o3})r_{o1} \]
  - Output resistance boosted by intrinsic gain of \( M_3, g_{m3}r_{o3} \)
- But how do we reduce the influence of large signal \( V_{ds} \) mismatch between \( M_1 \) and \( M_2 \)?
Match $V_{ds}$ of Current Mirror Devices With Proper Bias

- **Key transistor for determining** $I_2$ **is** $M_1$
  - Why is $M_2$ less important?
- **Above biasing approach provides a much closer match** between $V_{ds1}$ and $V_{ds4}$

\[
I_2 = \frac{W_1}{W_4} \frac{1 + \lambda V_{ds1}}{1 + \lambda V_{ds4}} I_1 \approx \frac{W_1}{W_4} I_1
\]

Recall:
\[
\frac{I_2}{I_1} = \frac{W_1}{W_4} \frac{L_4}{L_1} \frac{(1 + \lambda_1 V_{ds1})}{(1 + \lambda_4 V_{ds4})}
\]

- Current setting based on geometry
- Mismatch due to $V_{ds}$ difference
The Drawback of Basic Cascode Bias Approach

- Output voltage range is reduced
  - Now $V_o$ must be $> V_{TH} + 2\Delta V$
  - What will happen to the output impedance of the current source if the output voltage is too low?
  - Can we improve the voltage range?
**Improved Swing Cascode**

- **Key idea:** set size of $M_3$ such that $V_{ds1} = \Delta V$
  - Assuming strong inversion for $M_1$ and $M_3$:

$$\Delta V = \sqrt{\frac{2I_dL}{\mu n C_{ox} W}} \implies \alpha = \frac{1}{4}$$
Alternative Implementation of Improved Swing Cascode

- Set $\alpha$ as on previous slide
- Note: both implementations share a common problem
The Issue of Current Mismatch

- The improved swing approach causes a systematic mismatch between $I_2$ and $I_1$
  - Key issue: $V_{ds1} \neq V_{ds4}$

- Can we fix this problem?

Recall: \[ \frac{I_2}{I_1} = \frac{W_2}{W_1} \frac{(1+\lambda_2 V_{ds2})}{(1+\lambda_1 V_{ds1})} \]

Mismatch due to $V_{ds}$ difference
Techniques to Reduce Current Mismatch

- Systematic mismatch between $I_1$ and $I_2$ is greatly reduced by using the above circuit (now $V_{ds1} \approx V_{ds4}$)
  - Note that gate bias on $M_2$ and $M_3$ may be provided by previously discussed circuits

- Additional techniques for accurately matching $I_1$ and $I_2$
  - Set $L_1 = L_4 \gg L_{\text{min}}$
    - Note: set $L_2 = L_3 \approx L_{\text{min}}$ for lower area and capacitance
  - Set $W_2/W_3 = I_2/I_1$ so that $\Delta V_2 = \Delta V_3$
Another Common Cascode Bias Topology

- Key issue: needs two bias current branches
Utilizing a Simple Resistor to Achieve One Bias Branch

- Issue: poly resistor is large and won’t track NMOS devices across temperature and process variations
Better Approach: Use PMOS Device In Triode Region

- Much smaller, better tracking with NMOS devices than resistor
**Wilson Current Mirror**

- Relies on feedback in its operation
- Using Hybrid-\(\pi\) analysis

\[
R_{thd2} \approx \frac{1}{g_{m1}} (g_{m2}r_{o2}) (g_{m3}r_{o3})
\]

- Output resistance comparable to cascode current source
- This circuit is rarely used these days
Enhanced Cascode Current Source

- Offers output resistance comparable to double cascode current source
- As with Wilson mirror, analysis is tricky due to source/gate coupling
  - Using results shown in the following slide:
    \[ R_{thd4} \approx (g_{m4}r_{o4})(g_{m3}r_{o3})r_{o1} \]
Thevenin Resistances for CMOS Transistor Feedback Pair

\[ R_{thd} = r_{o4} \left( 1 + \left( g_{m4} \left( 1 + g_{m3} \left( R_A \| r_{o3} \right) \right) \right) + \frac{1}{r_{o4} + g_{mb4}} \right) R_B \]

\[ \approx \left( g_{m4} r_{o4} \right) \left( g_{m3} \left( r_{o3} \| R_A \right) \right) R_B \]

\[ R_{ths} = \left( 1 + \frac{R_C}{r_{o4}} \right) \left( \frac{1}{g_{mb4} r_{o4}} \right) \left( \frac{1}{g_{m4} \left( 1 + g_{m3} \left( r_{o3} \| R_A \right) \right)} \right) \]

\[ \approx \left( 1 + \frac{R_C}{r_{o4}} \right) \frac{1}{g_{m4} \left( g_{m3} \left( r_{o3} \| R_A \right) \right)} \]
Basic Cascode Amplifier

- Allows improved frequency response (discussed later)
- Reduction to two-port will be done in several steps
Eliminate Middle Sections

- Calculation of $G_{m1}$ same as for common source amp
- To reduce further, note that

$$R_{th_{d1}} \gg R_{th_{s2}} \implies \alpha_2 i_{s2} = i_{s2} \approx G_{m1}v_{g1}$$
**Key difference: drain impedance much larger**

\[
R_{thd2} \approx r_o2(1+g_m2R_{thd1}) \approx r_o2(1+g_m2r_o1(1+g_{m1}R_s)) \\
\approx (g_m2r_o2)(g_{m1}r_o1)R_s
\]
Slight Twist to Cascode Amplifier

- What is the difference between this amplifier and basic cascode amplifier?
- What are the constraints in setting $V_{bias}$?
- What is the maximum output voltage swing?
### Constraints on $V_{bias}$ and Output Range

- **To keep $M_2$ and $M_4$ in saturation**

  $$V_{bias} - (V_{TH} + \Delta V_1) > \max(\Delta V_2, \Delta V_4)$$

  $$\Rightarrow V_{bias} > V_{TH} + \Delta V_1 + \max(\Delta V_2, \Delta V_4)$$

- **To keep $M_1$ in saturation**

  $$V_{out} - (V_{bias} - (V_{TH} + \Delta V_1)) > \Delta V_1$$

  $$\Rightarrow V_{out} > V_{bias} - V_{TH}$$
**Calculation of Maximum Output Range**

- Minimum $V_{bias}$ allows the maximum output range

\[ \Rightarrow V_{bias} = V_{TH} + \Delta V_1 + \max(\Delta V_2, \Delta V_4) \]

- Resulting output range

\[ V_{bias} - V_{TH} < V_{out} < V_{dd} \]
\[ \Delta V_1 + \max(\Delta V_2, \Delta V_4) < V_{out} < V_{dd} \]
We can turn the enhanced cascode current source into an amplifier
- Inject a current input at the source of $M_4$

Key aspects of small signal analysis can be done using Thevenin method
- Simply leverage Thevenin resistance formulas shown on Slide 16
Small-Signal Analysis of Enhanced Cascode Amp

From Thevenin resistance calculations on Slide 16:

- Input impedance is quite low

\[
R_{in} \approx R_{thd1} \parallel \left( 1 + \frac{R_1}{r_{o4}} \right) \frac{1}{g_{m4}(g_{m3}r_{o3})} \approx \frac{1}{g_{m4}(g_{m3}r_{o3})}
\]

- Output impedance is probably determined by \( R_1 \)

\[
R_{out} \approx R_1 \parallel (g_{m4}r_{o4})(g_{m3}r_{o3})(R_{thd1} \parallel R_s) \approx R_1
\]

- This amplifier is useful for extracting a current signal while keeping the source voltage nearly constant