Analysis and Design of Analog Integrated Circuits
Lecture 2

Two-Port Models, Frequency Response

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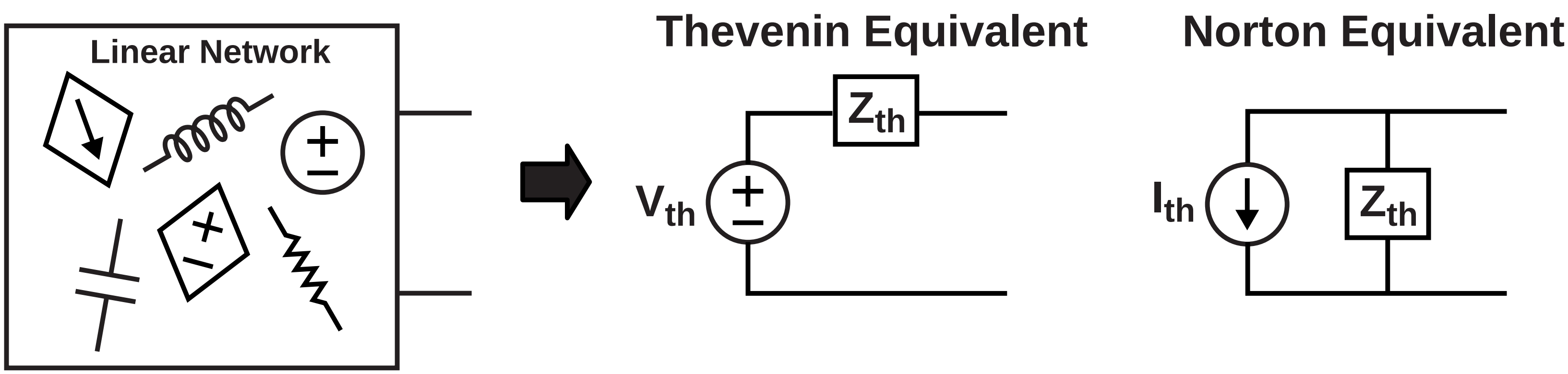
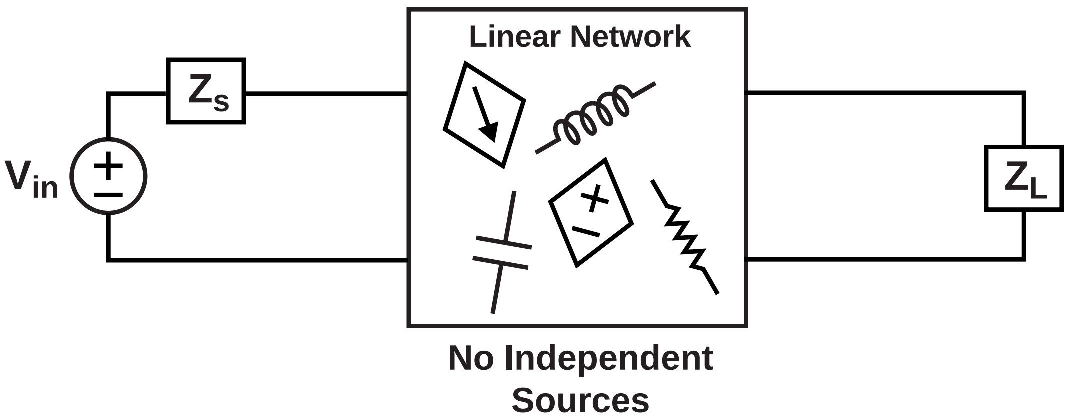
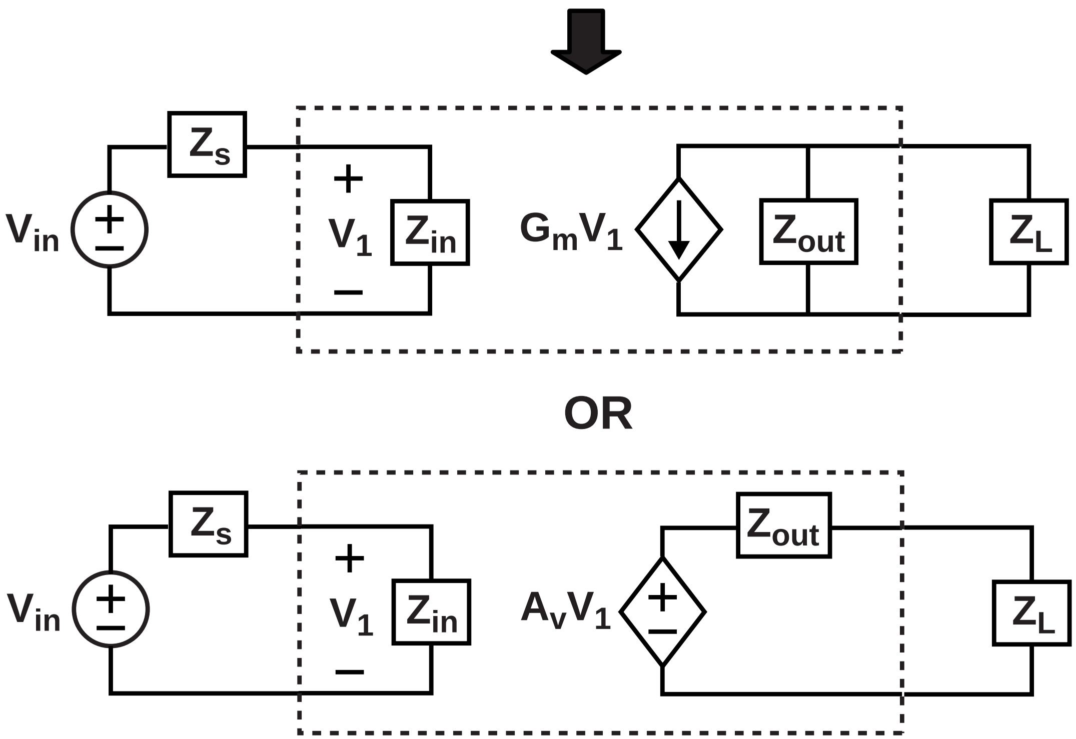
Review: Basics of One-Port Modeling



- V_{th} computed as open circuit voltage at port nodes
- I_{th} computed as short circuit current across port nodes
- Z_{th} computed as V_{th}/I_{th}
  - All independent voltage and current sources are set to zero value

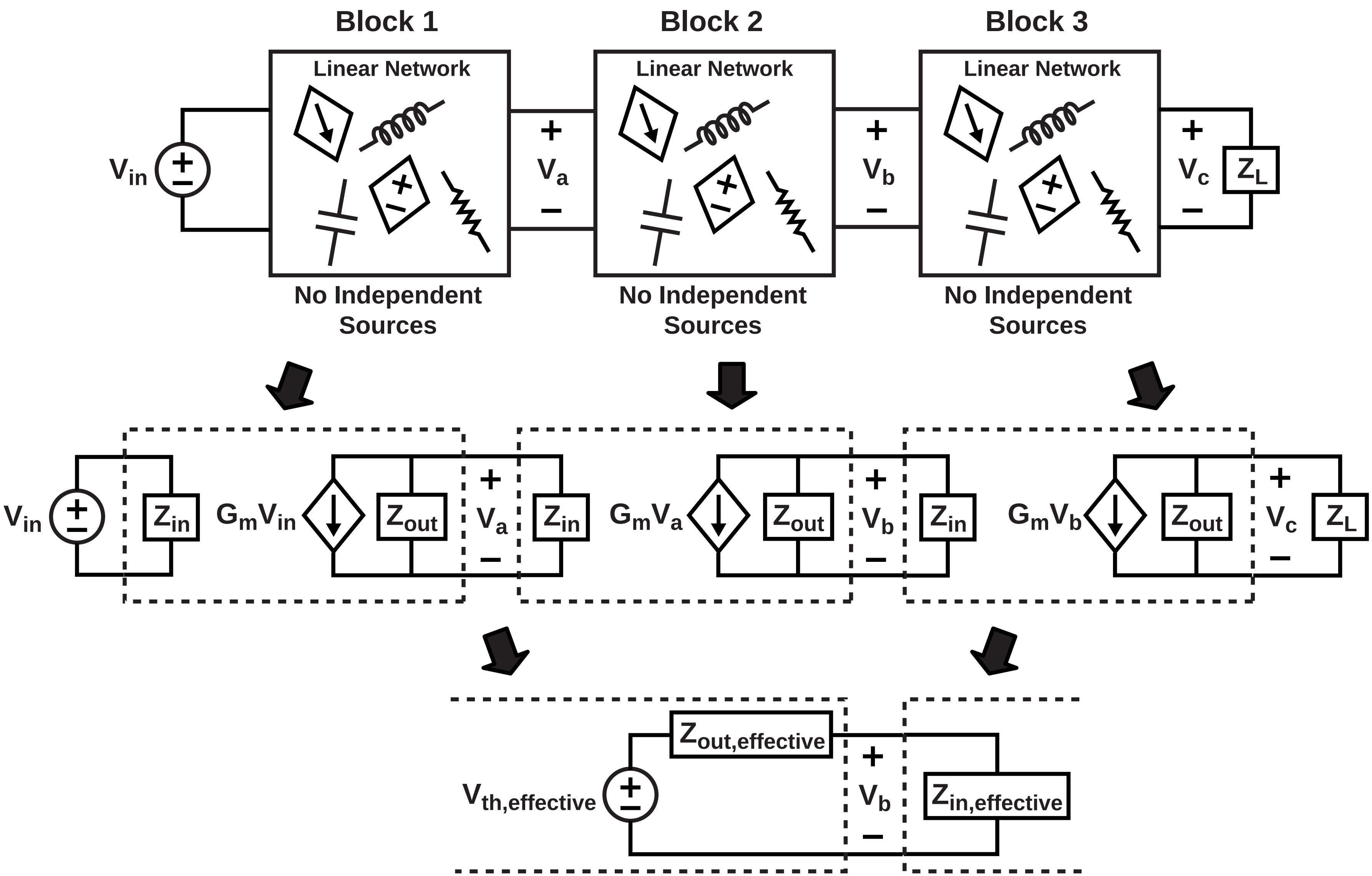
Basics of Two-Port Modeling (Unilateral)





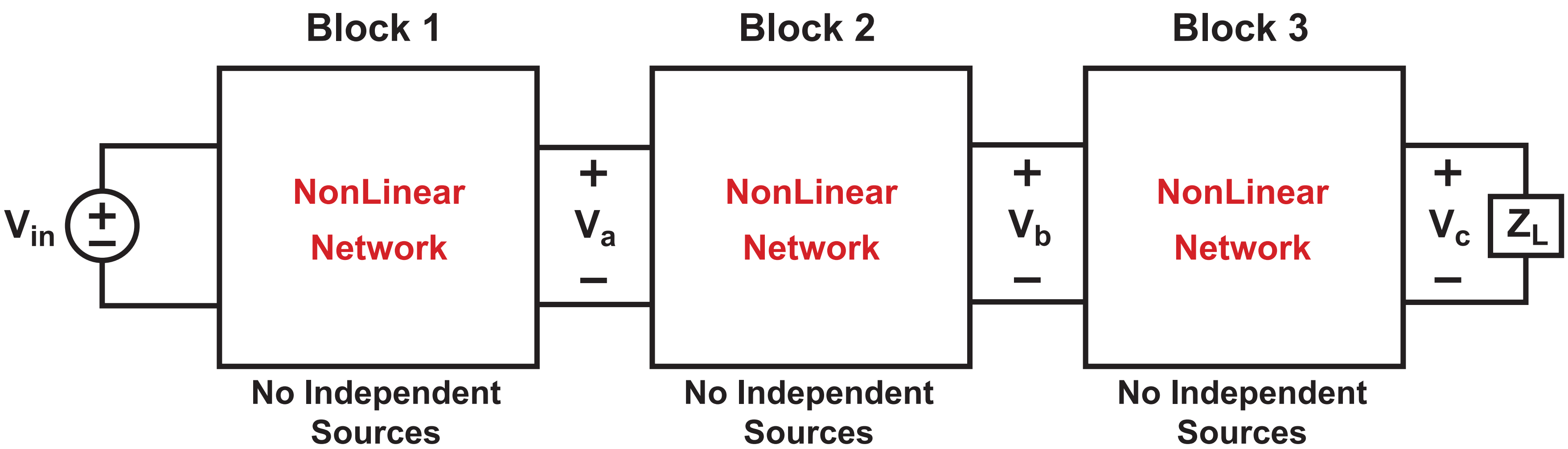
- We now include a dependent current or voltage source
- Z_{in}
  - Solve using 1-Port analysis at input
- Z_{out}
  - Solve using 1-Port analysis at output with $V_1 = 0$
- G_M
  - Short circuit output current as a function of V_1
- A_v
  - Open circuit output voltage as a function of V_1

Analysis of Cascaded Blocks



Analysis carried out without solving simultaneous equations!

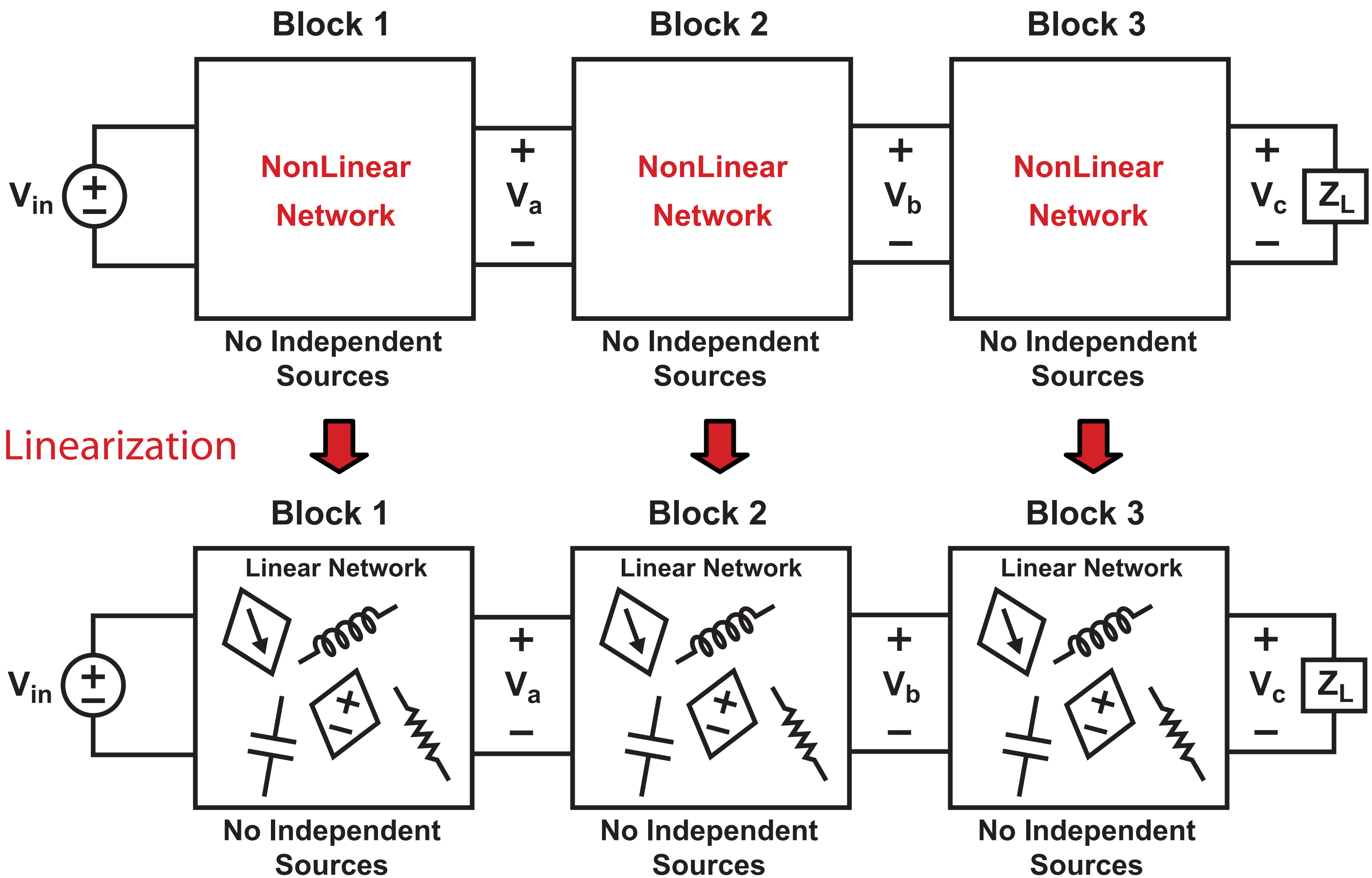
Problem: Most Circuits are Very Nonlinear!



- **Thevenin/Norton modeling only applies to linear networks**
- **Direct analysis of nonlinear networks is challenging**

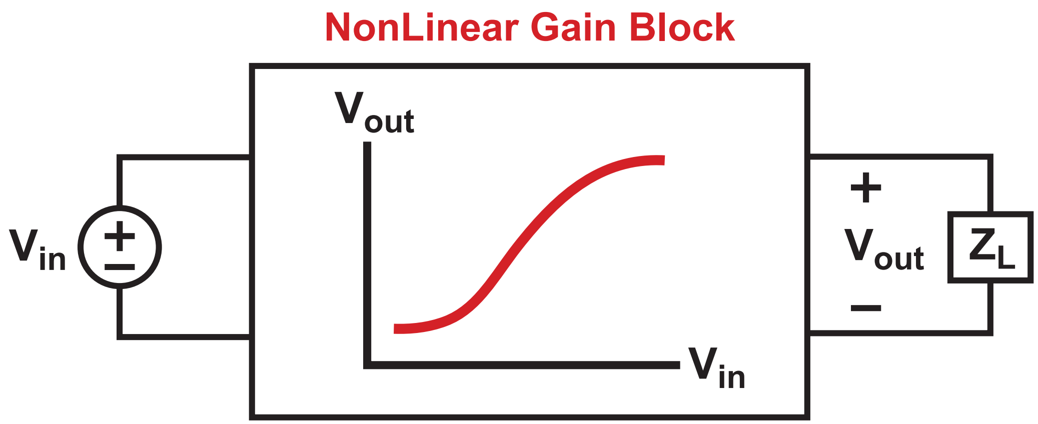
Can we still leverage two-port modeling?

Small Signal Modeling Allows Us to Linearize



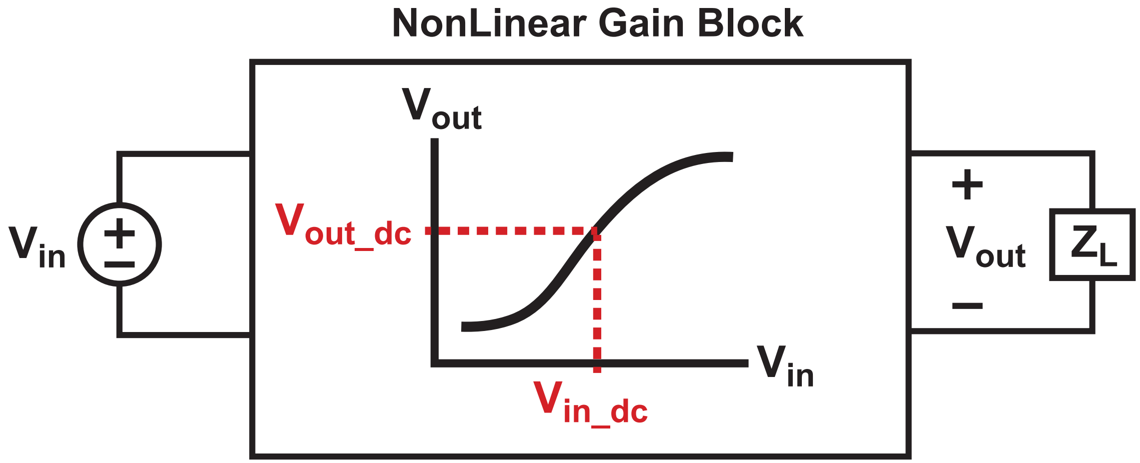
Small signal model is only valid about a specific operating point

Small Versus Large Signal Modeling



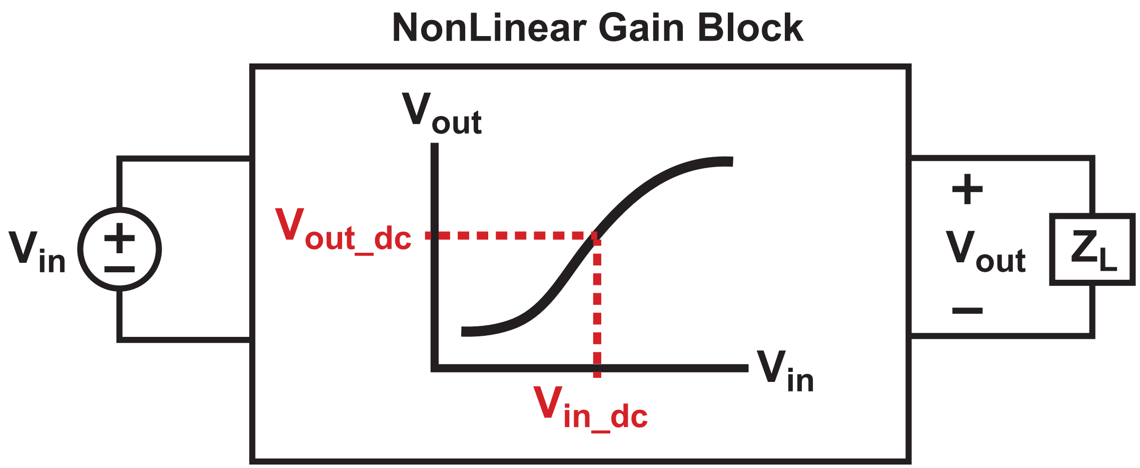
- **Sketch V_{out} versus V_{in} as the amplitude of V_{in} is increased**

Impact of Operating Point on Small Signal Modeling



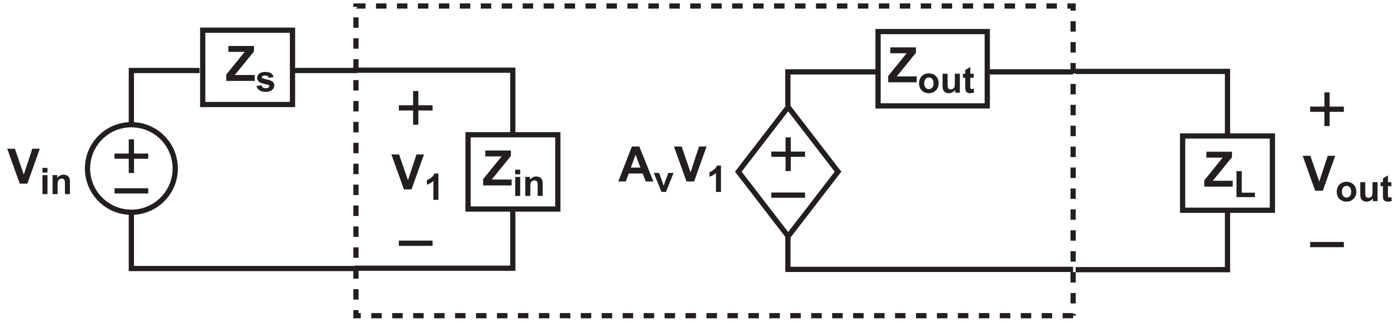
- **Sketch V_{out} versus V_{in} as the DC operating point is changed**

Achieving a Small Signal Model



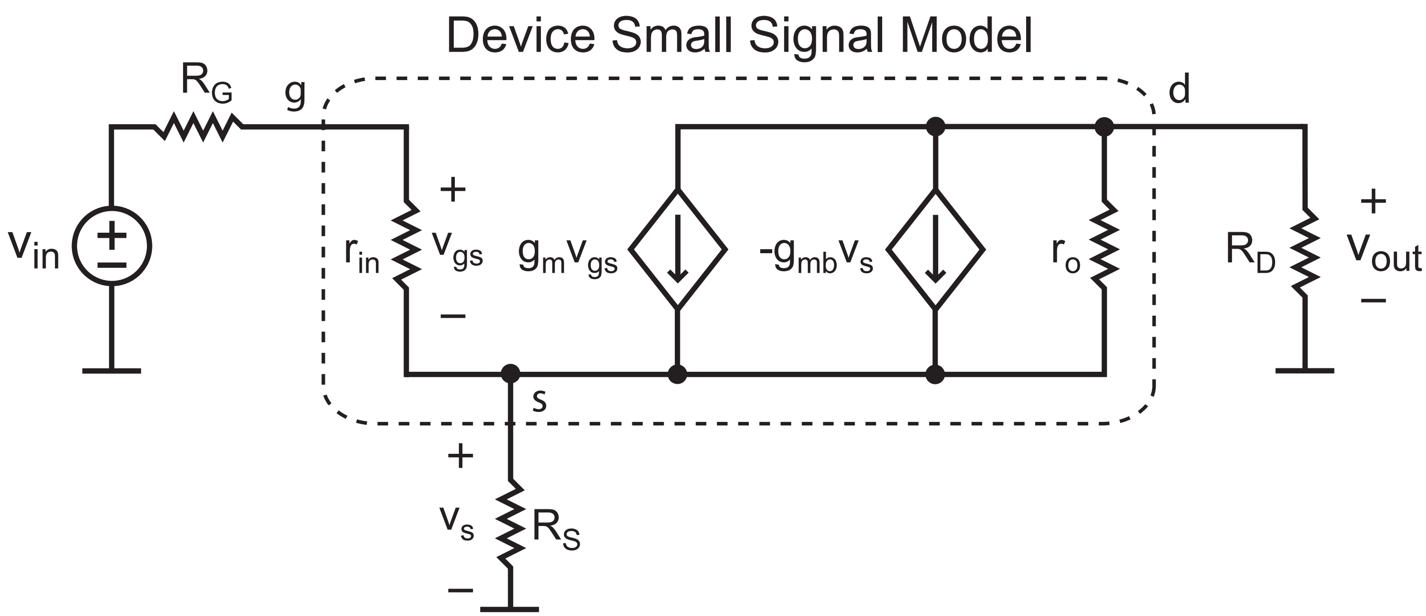
- **Create a two port model of the above block**

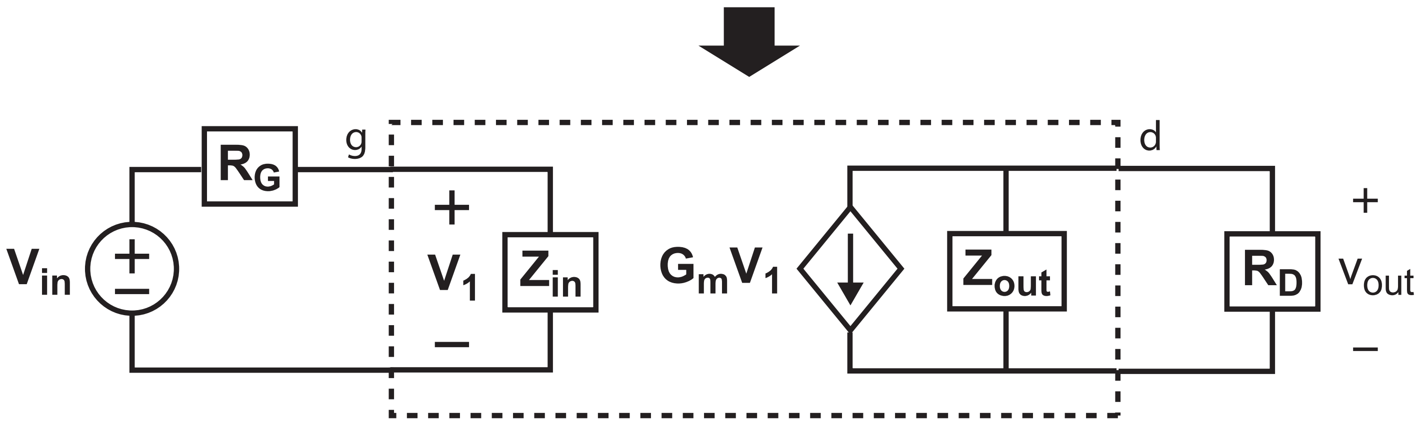
Including Impedances in Two-Port Models



- **Compute V_{out} as a function of V_{in}**

Example of Two-Port Derivation

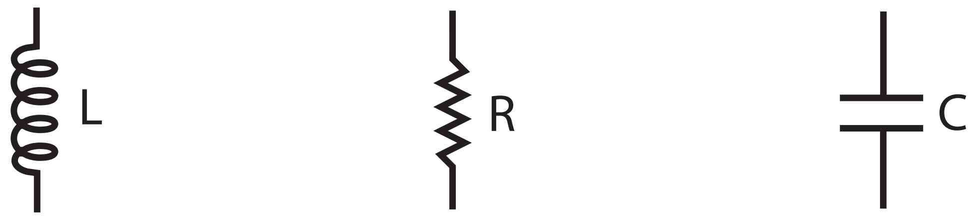




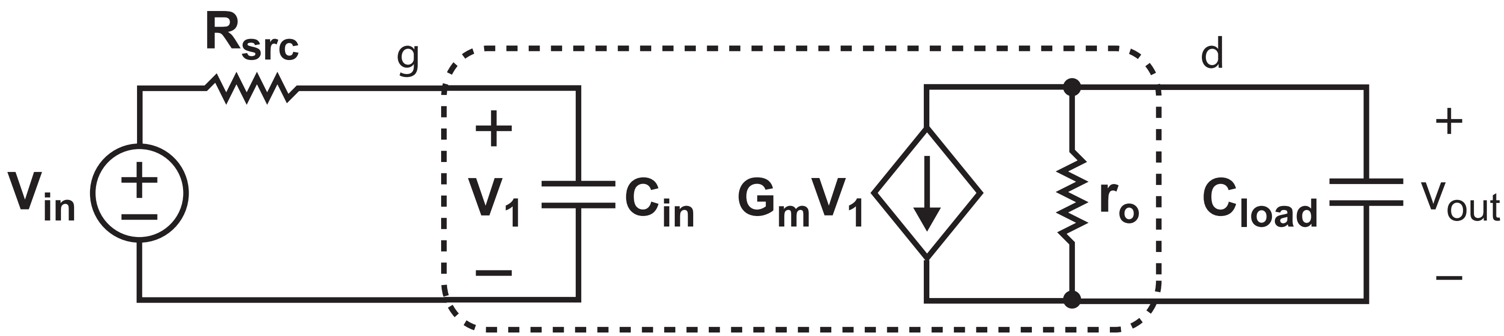
- Compute Z_{in}, Z_{out}, and G_m
  - Assume r_{in} = infinity, $g_{mb} = 0$

Frequency Domain Modeling of Impedances

- **Determine Laplace Transform of Impedances Below:**



Example: Transfer Function of Two-Port Circuit



- Derive the transfer function $V_{out}(s)/V_{in}(s)$
- Label the poles and zeros of the transfer function

Frequency Response

- **Frequency response is readily derived from a transfer function:**
  - **For *w* (rad/s), you substitute *s* = *jw***
  - **For *f* (Hz), you substitute *s* = *j2πf***
  - **Note that *j* = sqrt(-1)**
- **Example, for the transfer function on the previous page, the frequency response (in *f* (Hz)) is:**

Bode Plot Basics

- **The magnitude and phase of the frequency response is often depicted in the form of a Bode plot**
- **Example:** $$H(w) = \frac{V_{out}(w)}{V_{in}(w)} = \frac{1 + jw/w_z}{(1 + jw/w_{p1})(1 + jw/w_{p2})}$$
  - **Log of magnitude (dB):** $20 \log |H(w)|$

    $$= 20 \log |1 + jw/w_z| - 20 \log |1 + jw/w_{p1}| - 20 \log |1 + jw/w_{p2}|$$

    - Taking the log allows the poles and zeros to be plotted separately and then added together
  - **Phase:** $\angle H(w)$

    $$= \angle(1 + jw/w_z) - \angle(1 + jw/w_{p1}) - \angle(1 + jw/w_{p2})$$

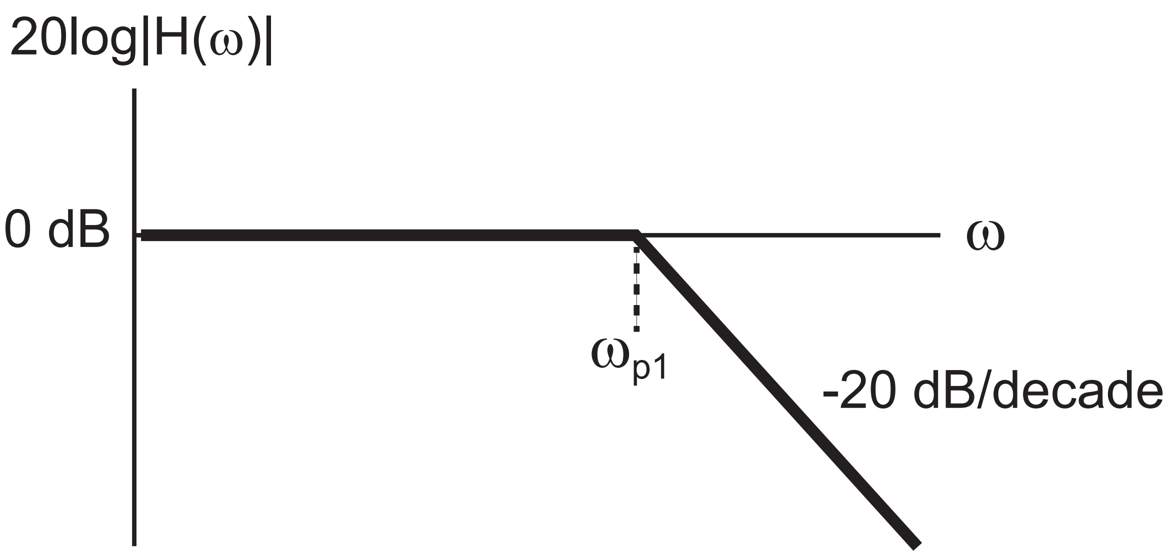
    - Phase of poles and zeros can also be plotted separately and then added together

Plotting the Magnitude of Poles

- **Plot the magnitude response of pole w_{p1}**

$$20\log|H(w)| = 20\log\left|\frac{1}{1+jw/w_{p1}}\right| = -20\log|1+jw/w_{p1}|$$

  - **For w << w_{p1}:** $20\log|H(w)| \approx -20\log|1| = 0$
  - **For w >> w_{p1}:** $20\log|H(w)| \approx -20\log|w/w_{p1}|$

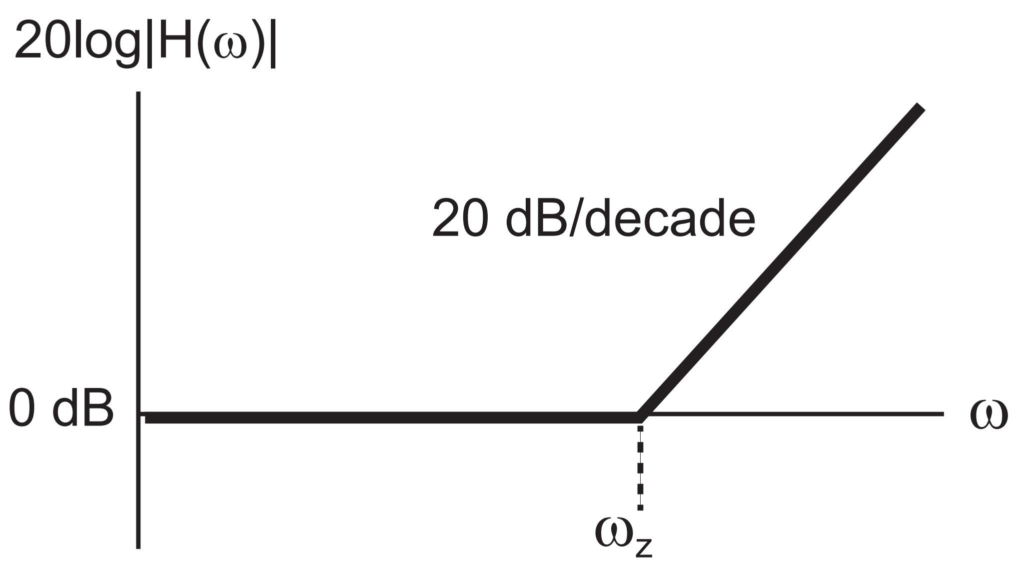


Plotting the Magnitude of Zeros

- **Plot the magnitude response of pole w_z**

$$20 \log |H(w)| = 20 \log |1 + jw/w_z|$$

  - **For $w << w_z$:** $$20 \log |H(w)| \approx 20 \log |1| = 0$$
  - **For $w >> w_z$:** $$20 \log |H(w)| \approx 20 \log |w/w_z|$$

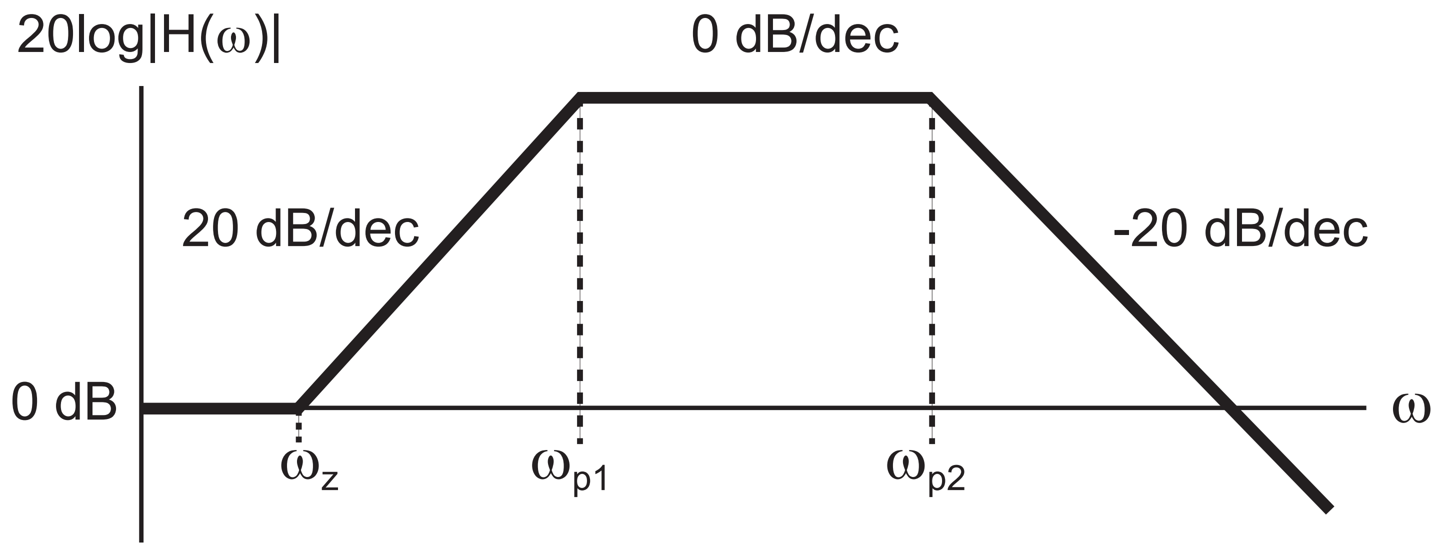


Putting It All Together

- **Example Frequency Response:**

$$H(w) = \frac{V_{out}(w)}{V_{in}(w)} = \frac{1 + jw/w_z}{(1 + jw/w_{p1})(1 + jw/w_{p2})}$$

  - **Assume $w_z << w_{p1} << w_{p2}$**



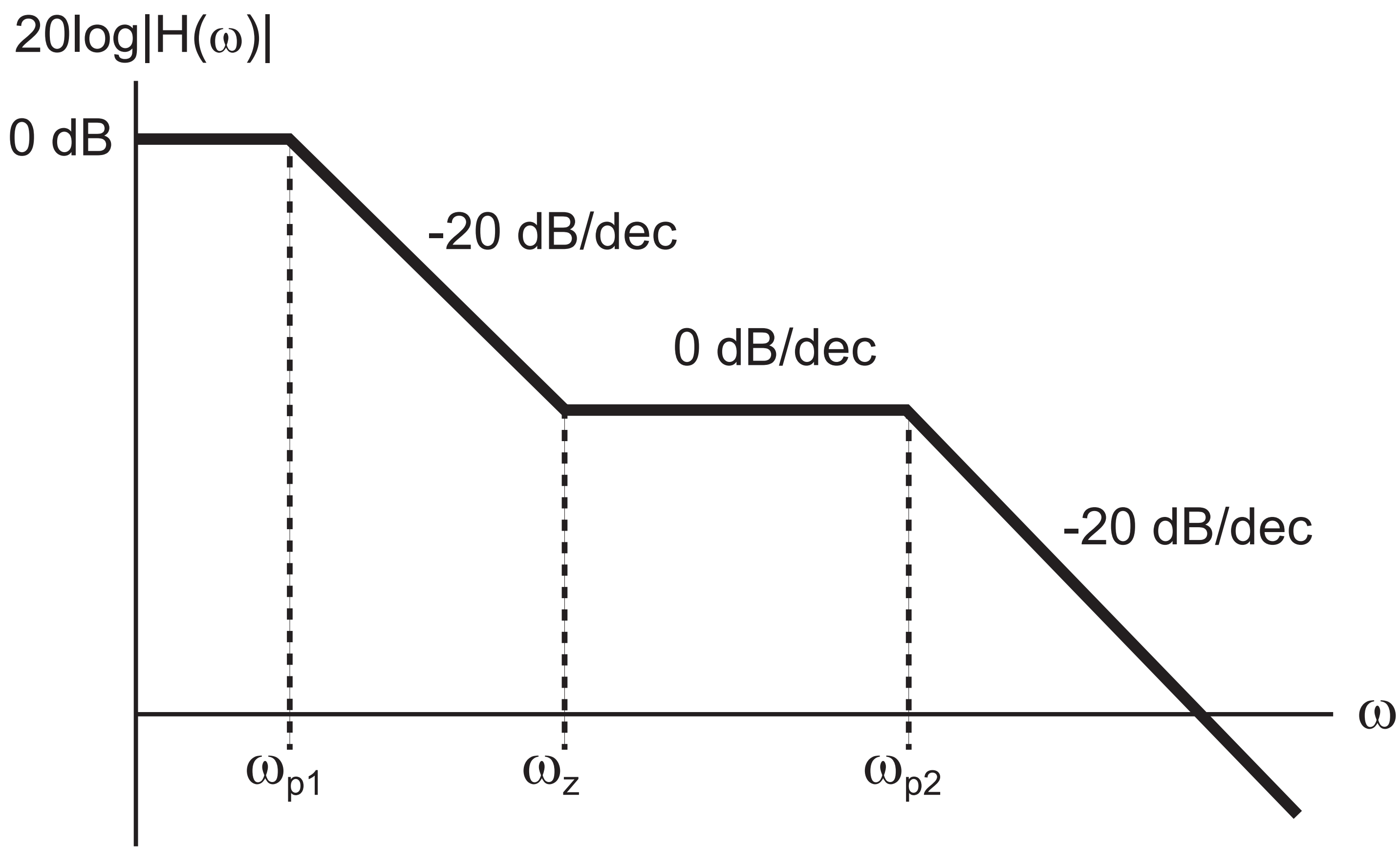
- **What happens if $w_{p1} << w_z << w_{p2}$?**

Changing the Order of Poles and Zeros

- **Example Frequency Response:**

$$H(w) = \frac{V_{out}(w)}{V_{in}(w)} = \frac{1 + jw/w_z}{(1 + jw/w_{p1})(1 + jw/w_{p2})}$$

  - **Assume $w_{p1} << w_z << w_{p2}$**



Changing the DC Gain from 1 to K

- **Example Frequency Response:**

$$H(w) = \frac{V_{out}(w)}{V_{in}(w)} = K\frac{1 + jw/w_z}{(1 + jw/w_{p1})(1 + jw/w_{p2})}$$

  - **Assume $w_{p1} << w_z << w_{p2}$**