I/Q Modulation and RC Filtering

- Issues with coherent modulation
- Analog I/Q modulation principles
- RC networks as continuous-time filters
- Differentiation property of Fourier Transform

Copyright © 2007 by M.H. Perrott
All rights reserved.
**AM Modulation and Demodulation**

- **Multiplication (i.e., mixing) operation shifts in frequency**
  - Also creates undesired high frequency components at receiver

- **Lowpass filtering passes only the desired baseband signal at receiver**

What can go wrong here?
Impact of 90 Degree Phase Shift

- If receiver cosine wave turns into a sine wave, we suddenly receive no baseband signal!
  - We apparently need to synchronize the phase of the transmitter and receiver local oscillators
    - This is called coherent demodulation

- Some key questions:
  - How do we analyze this issue?
  - What would be the impact of a small frequency offset?
Frequency Domain Analysis

- When transmitter and receiver local oscillators are matched in phase:
  - Demodulated signal *constructively* adds at baseband
Impact of 90 Degree Phase Shift

- When transmitter and receiver local oscillators are 90 degree offset in phase:
  - Demodulated signal destructively adds at baseband

What would happen with a small frequency offset?
I/Q Modulation

- Consider modulating with both a cosine and sine wave and then adding the results
  - This is known as I/Q modulation

- The I/Q signals occupy the same frequency band, but one is real and one is imaginary
  - We will see that we can recover both of these signals
I/Q Demodulation

- Demodulate with both a cosine and sine wave
  - Both I and Q channels are recovered!
- I/Q modulation allows twice the amount of information to be sent compared to basic AM modulation with same bandwidth

What can go wrong here?
Impact of 90 Degree Phase Shift

- **I and Q channels get swapped at receiver**
  - Key observation: no *information* is lost!

- **Questions**
  - What would happen with a *small* frequency offset?
  - What would happen with a *large* frequency offset?
Summary of Analog I/Q Modulation

- **Frequency domain view**

  Baseband Input

  \[ i_I(t), Q_I(t) \]

  \[ f \]

  \[ 0 \]

  \[ 1 \]

  Receiver Output

  \[ i_R(t), Q_R(t) \]

  \[ f \]

  \[ 0 \]

  \[ 1 \]

- **Time domain view**

  Baseband Input

  \[ i_I(t), Q_I(t) \]

  \[ t \]

  \[ 0 \]

  \[ 1 \]

  Receiver Output

  \[ i_R(t), Q_R(t) \]

  \[ t \]

  \[ 0 \]

  \[ 1 \]
• Analyze by first deriving a differential equation relating output and input voltages

\[ I_R(t) = \frac{V_{in}(t) - V_{out}(t)}{R} = C \frac{dV_{out}(t)}{dt} \]

• The filter frequency response is defined as

\[ H(f) = \frac{V_{out}(f)}{V_{in}(f)} \]

• The output voltage corresponds to a scaled and phase shifted version of the input cosine wave

\[ V_{out}(t) = |H(f_o)| \cos(2\pi f_o t + \angle H(f_o)) \]
Differentiation Property of FT

Fourier Transform Definition

\[
x(t) \iff X(f)
\]

\[
x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft} df
\]

\[
X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt
\]

- Derive impact of differentiation

\[
\frac{d}{dt} x(t) = \frac{d}{df} \int_{-\infty}^{\infty} X(f)e^{j2\pi ft} df
\]

\[
= \int_{-\infty}^{\infty} j2\pi f X(f)e^{j2\pi ft} df
\]

\[
\frac{d}{dt} x(t) \iff j2\pi f X(f)
\]
Derivation of RC Filter Response

\[ V_{in}(t) - V_{out}(t) = R \frac{dV_{out}(t)}{dt} \]

- Apply FT to above differential equation

\[ \frac{V_{in}(f) - V_{out}(f)}{R} = C j2\pi f V_{out}(f) \]

\[ \Rightarrow \frac{V_{in}(f)}{R} = \left( C j2\pi f + \frac{1}{R} \right) V_{out}(f) \]

- Filter frequency response is then calculated as

\[ H(f) = \frac{V_{out}(f)}{V_{in}(f)} = \frac{1}{1 + RC j2\pi f} \]
Magnitude of RC Filter Response

- Define cutoff frequency of filter
  \[ f_c = \frac{1}{2\pi RC} \quad \Rightarrow \quad H(f) = \frac{1}{1 + j\frac{f}{f_c}} \]

- Magnitude of response:
  \[
  |H(f)| = \frac{1}{\sqrt{1 + (\frac{f}{f_c})^2}}
  \]

- Cases:
  \[
  \begin{align*}
  f &= f_c \quad \Rightarrow \quad |H(f)| = \frac{1}{\sqrt{2}} \\
  f &< f_c \quad \Rightarrow \quad |H(f)| \approx 1 \\
  f &> f_c \quad \Rightarrow \quad |H(f)| \approx \frac{f_c}{f}
  \end{align*}
  \]
Summary

- **Coherent modulation** requires synchronized local oscillators at transmitter and receiver
  - Impact of phase offset is to change baseband *amplitude*
  - Impact of frequency offset is *fading* (small offset) or catastrophic *corruption* (large offset) of baseband signal

- **I/Q modulation** allows twice the amount of information to be sent compared to basic AM
  - Impact of phase offset is to swap I/Q
  - Impact of frequency offset is I/Q swapping (small offset) or catastrophic corruption (large offset) of received signal

- **RC networks** provide *continuous-time* filtering

- **Upcoming lectures**
  - Examine another non-ideality: noise
  - Lay groundwork for *digital* modulation and the concept of *information*